

I

CONSIGLIO NAZIONALE  
DEGLI INGEGNERI



# L'utilizzo della dinamica sperimentale per la diagnostica e il monitoraggio delle strutture

WEBINAR – 5 dicembre 2022

## La dinamica sperimentale per il monitoraggio

Antonina Pirrotta

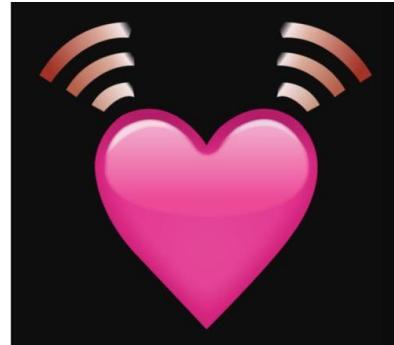
Università degli Studi di Palermo

Presidente IDEA

# Innovative Dynamics Experiments Association (IDEA)

- **Gianni BARTOLI**, Università di Firenze
- **Luigi CARASSALE**, Università di Genova
- **Giovanni FABBROCINO**, Università del Molise
- **Carmelo GENTILE**, Politecnico di Milano
- **Massimiliano GIOFFRÈ**, Università di Perugia
- **Giacomo Camillo NAVARRA**, Università Kore di Enna
- **Antonina PIRROTTA**, Università di Palermo
- **Walter SALVATORE**, Università di Pisa
- **Filippo UBERTINI**, Università di Perugia

**La Dinamica  
Sperimentale**



**cuore pulsante dell'Ingegnere**



**Monitoraggio**

Monitoraggio



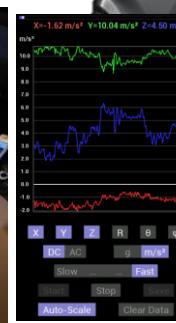
Steri di Palermo (Rettorato)



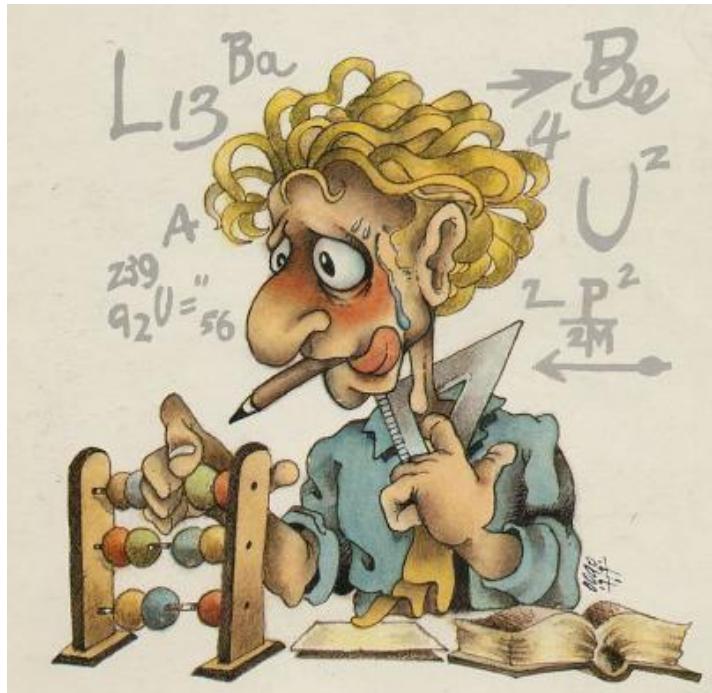
Radar



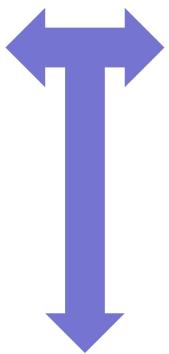
CROWDSENSE



# Mathematics

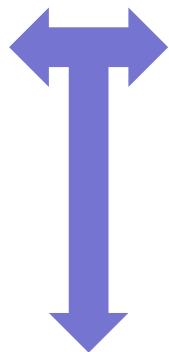


# Reality



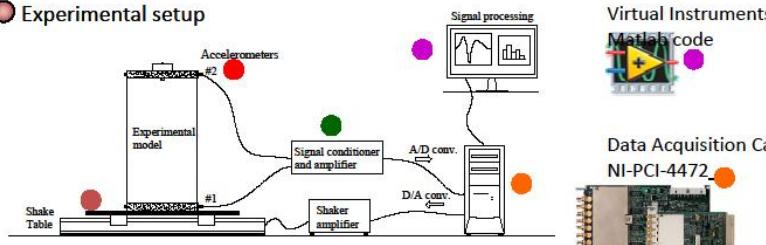
# Mathematics

# Reality



## Experimental Test

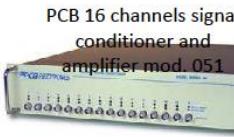
### • Experimental setup



Quanser ShakeTableII  
1DOF shaking table  
45.5x45.5 cm



Brüel&Kjær  
piezoelectric  
accelerometer  
mod. 4507-002B  
(1000mV/g sensitivity)



NI Labview  
Virtual Instrument:  
Matlab code



# Scientific Method Components

## Characterizations (observations)

Hypotheses (theoretical)

Predictions (reasoning )

Experiments



Galileo Galilei  
Pisa, 15 febbraio 1564 –  
Arcetri, 8 gennaio 1642

# Scientific Method Components

**Characterizations (observations)**

**Hypotheses (theoretical)**

**Predictions (reasoning )**

**Experiments**

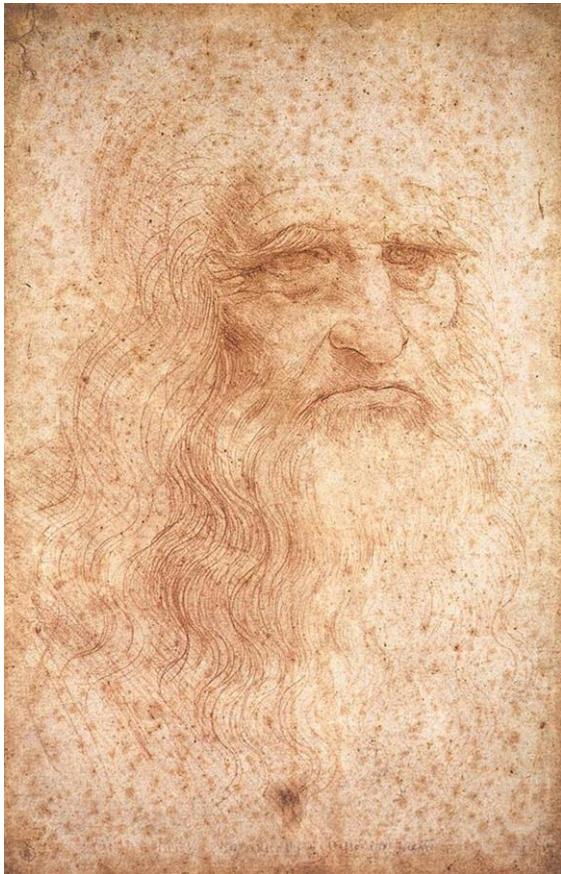


## **VIRTUAL EXPERIMENT**

Efficient Experiments allow to build up virtual experiments

Once a proposed theoretical formulation is reliable  
having confirmed efficient tests

Simulations from this Mathematical formulation  
are  
virtual experiments



Leonardo Da Vinci

Anchiano, 15 april 1452  
– Amboise, 2 may 1519

Those who fall in love with practice without science  
are like the helmsman, who enters a ship without a  
rudder or compass, who is never sure where to go.

Quelli che s'innamoran di pratica sanza scienzia son come 'l nocchier  
ch'entra in navilio senza timone o bussola, che mai ha certezza dove si  
vada. Sempre la pratica deve essere edificata sopra la bona teorica.

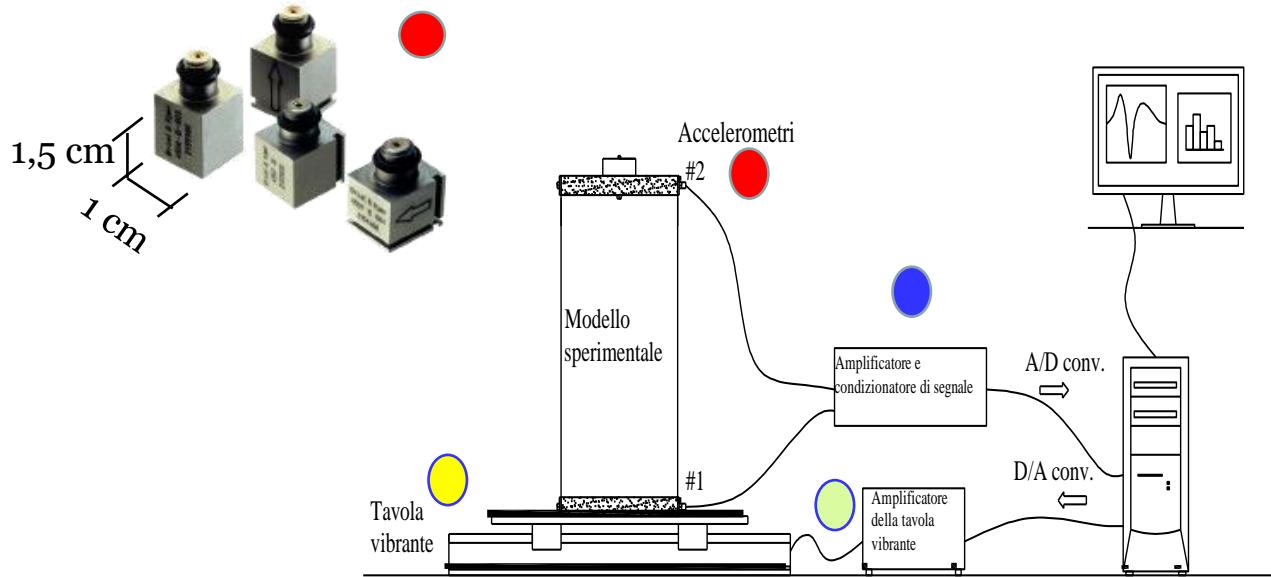
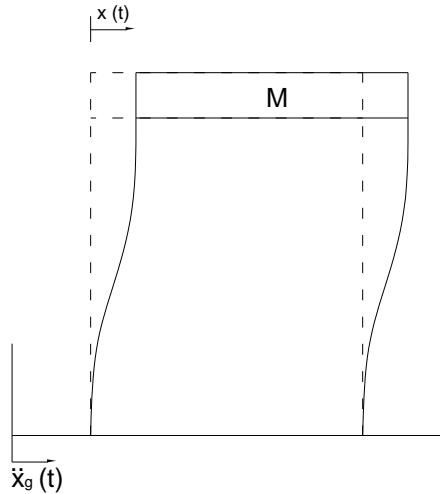
It doesn't matter how beautiful your theory is,  
it doesn't matter how smart you are.  
If it doesn't agree with experiment, it's wrong

Non importa quanto sia bella la tua teoria, non importa quanto tu sia  
intelligente. Se non concorda con l'esperimento, è sbagliata.



Richard Phillips Feynman  
11 May 1918 New York  
15 february 1988 Los Angeles

# Setup sperimentale



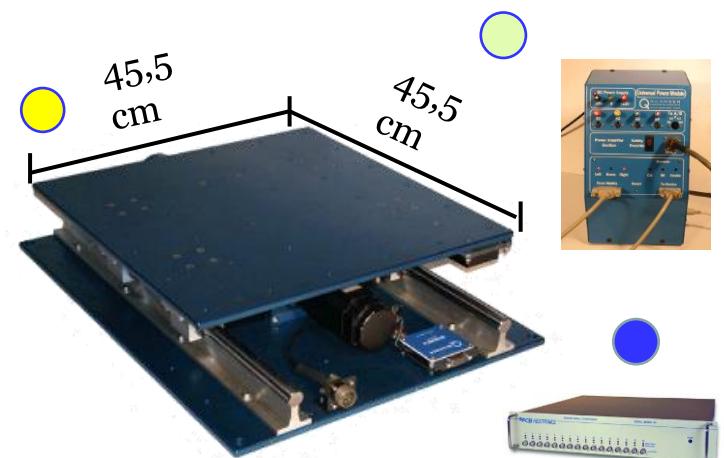
Modellazione matematica:  
il sistema non controllato  
(SDOF)

$$M\ddot{x} + C\dot{x} + Kx = -M\ddot{x}_g$$

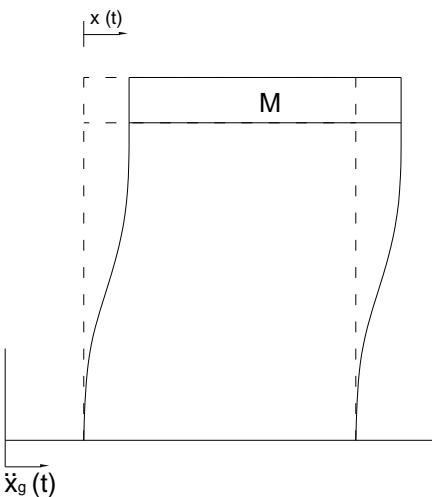
$$M = 4.503 \text{ kg};$$

$$C = 0.253 \text{ Ns / m};$$

$$K = 451.78 \text{ N / m}$$



# Setup sperimentale



Modellazione matematica:  
il sistema non controllato  
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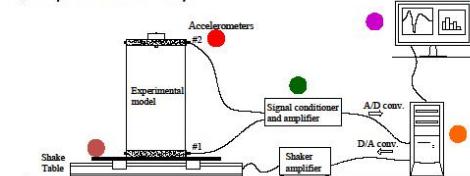
$$C = 0.253 \text{ Ns/m};$$

$$K = 451.78 \text{ N/m}$$

## Experimental Test

Experimental dynamic Lab of University of Palermo

● Experimental setup



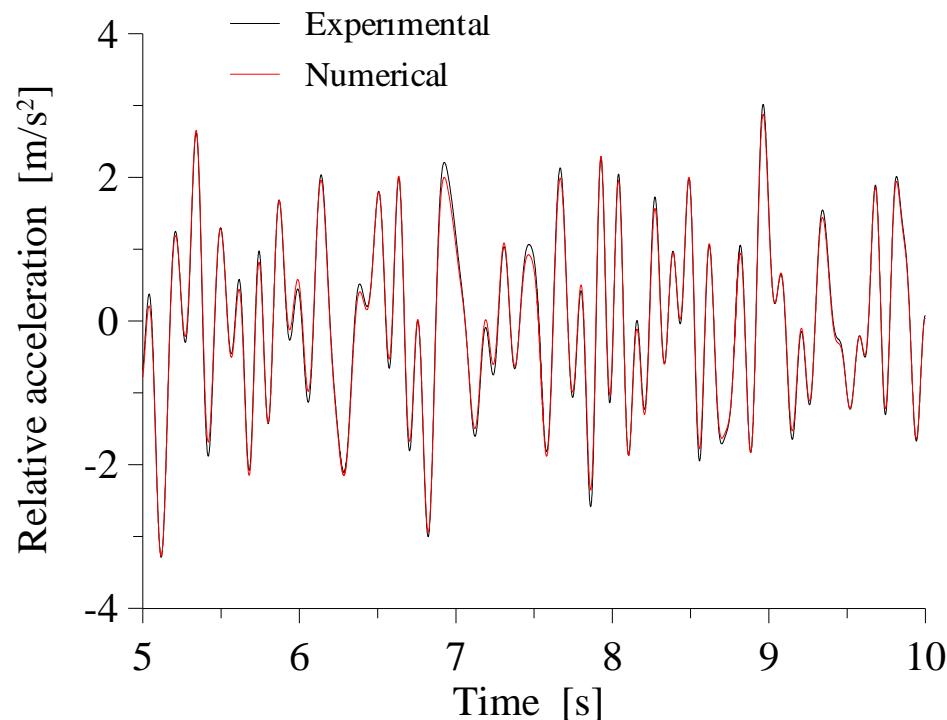
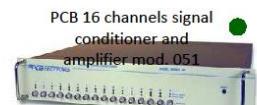
NI Labview  
Virtual Instrument:  
Matlab code



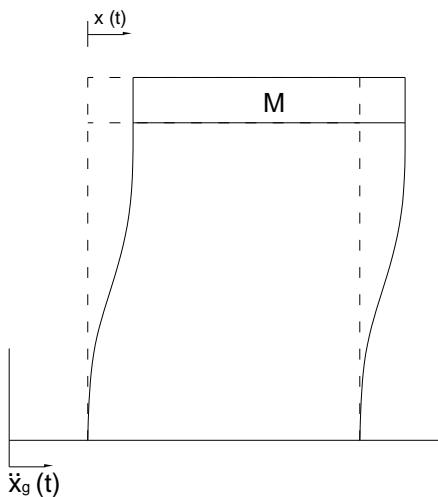
Quanser ShakeTableII  
1DOF shaking table  
45.5x45.5 cm



Brüel&Kjær  
piezoelectric  
accelerometer  
mod. 4507-002B  
(1000mV/g sensitivity)



# Setup sperimentale



Mathematical model

$$M\ddot{x} + C\dot{x} + Kx = -M\ddot{x}_g$$

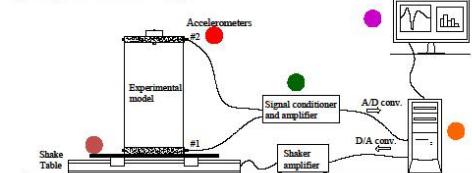
VIRTUAL

EXPERIMENT!!!!

## Experimental Test

Experimental dynamic Lab of University of Palermo

● Experimental setup



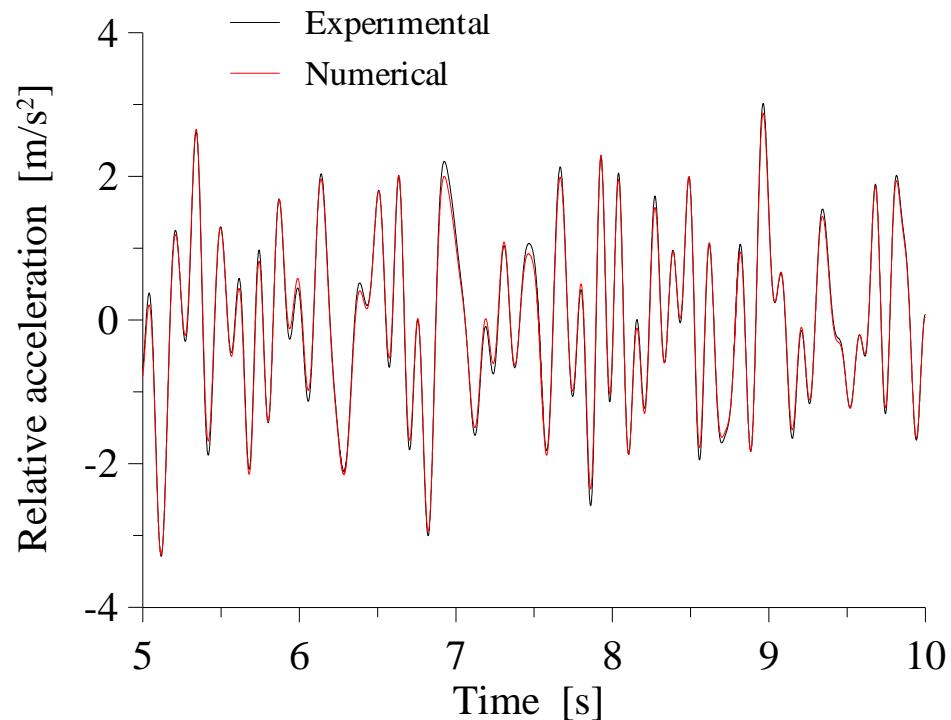
NI Labview  
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Quanser ShakeTable II  
1DOF shaking table  
45.5x45.5 cm



Brüel&Kjær  
piezoelectric  
accelerometer  
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(1000mV/g sensitivity)





UNIVERSITÀ  
DEGLI STUDI  
DI PALERMO

dij  
dipartimento  
di ingegneria  
unipa

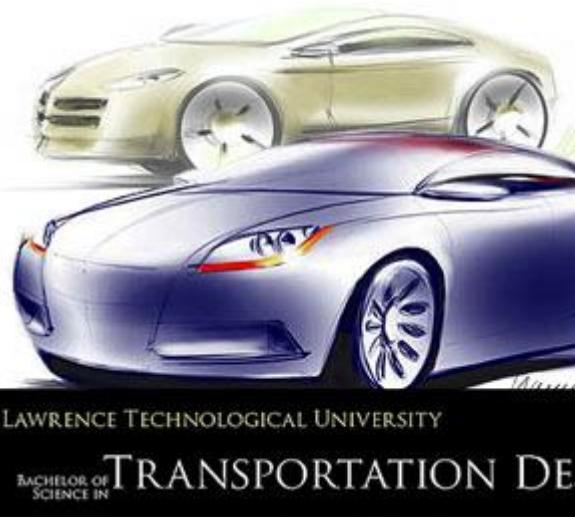
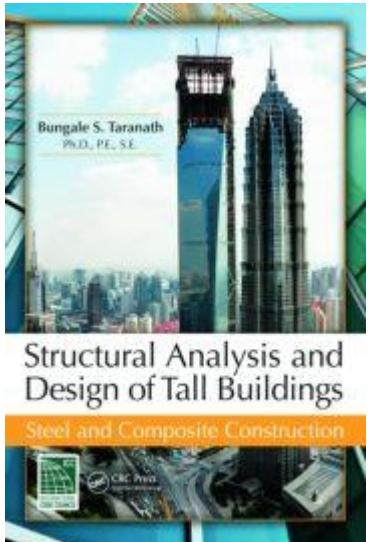
Sponsorizzato da



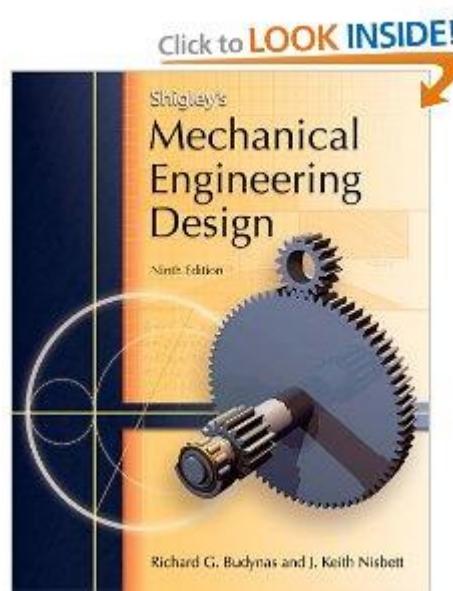
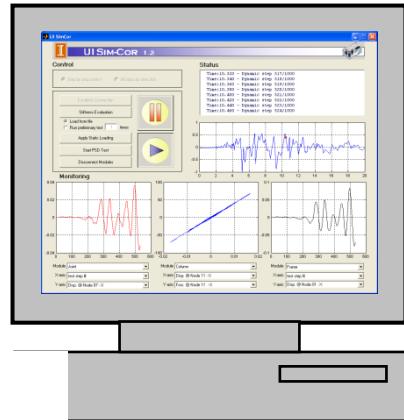
Progettato per l'uso non commerciale

Per rimuovere il fotogramma di Freemake, utilizza Gold Pack

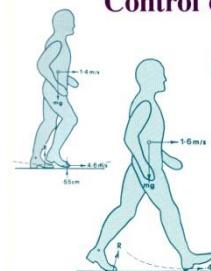
the most challenging problem



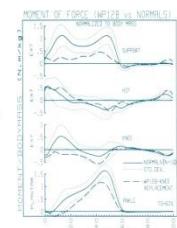
# Virtual Experiments



The Biomechanics and Motor Control of Human Gait:  
Normal, Elderly and Pathological



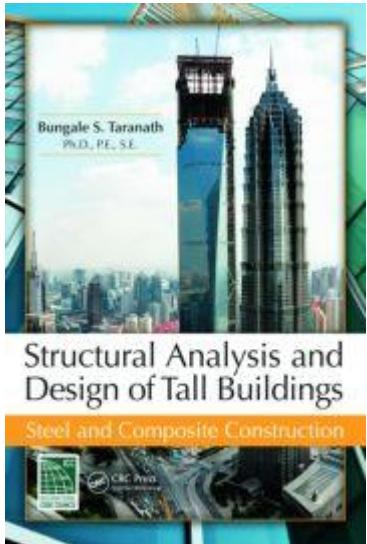
Second Edition



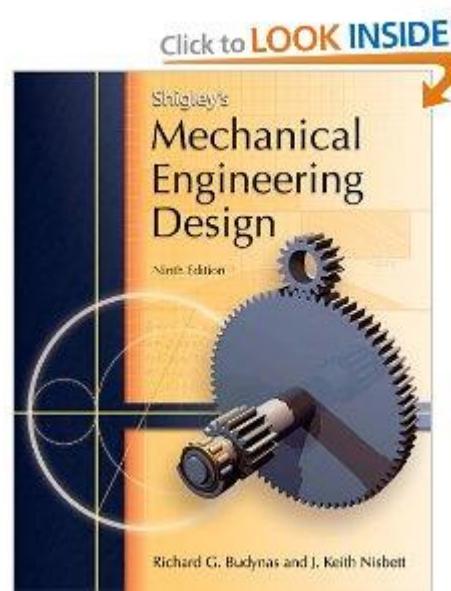
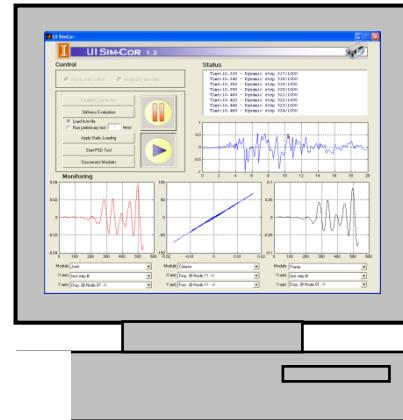
David A. Winter



the most challenging problem



# Virtual Experiments



Volume 20, Number 10, July 2014  
ISSN: 1077-5522

# JVC

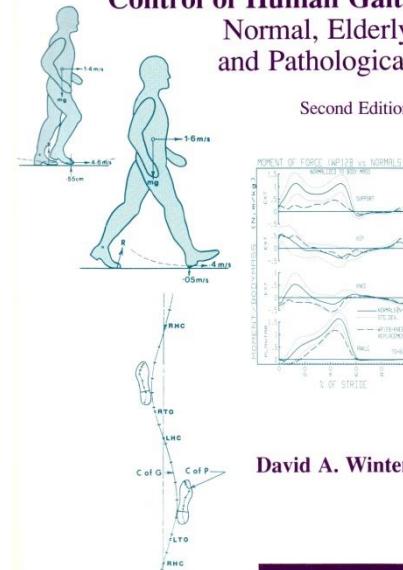
Journal of Vibration and Control

Editors  
Medhi Ahmadian  
Fabio Casciati  
Fabrizio Vestroni



The Biomechanics and Motor Control of Human Gait:  
Normal, Elderly and Pathological

Second Edition



David A. Winter

# Passive control systems

**A device is attached to a main system reducing vibration without external power supply**

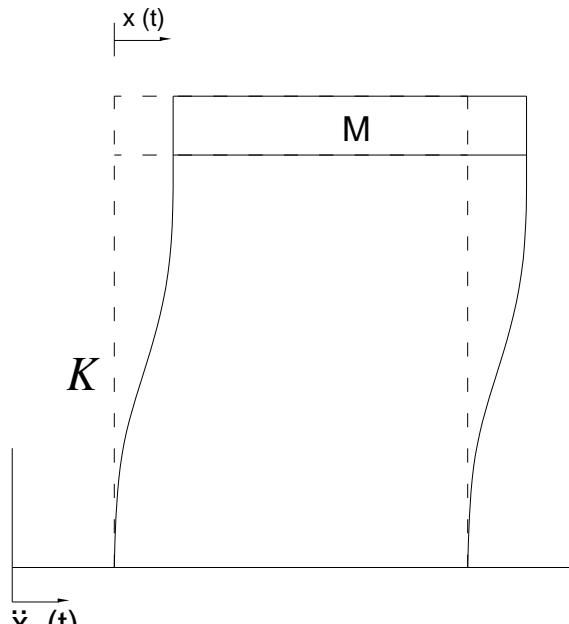
Ref.:  
 J.P. Den Hartog, 1928  
 "Mechanical vibration",  
 McGraw-Hill, New York

● Tuned Mass Damper (TMD)



Mass-spring-dashpot connected to the main system

Main system



$$M\ddot{x} + C\dot{x} + Kx = -M\ddot{x}_g$$

**TMD controlled system**

$$m_{TMD} = m$$

$$c = 2m\zeta_2\omega_{TMD}$$

$$k = m\omega_{TMD}^2$$

$$\omega_{TMD} = \sqrt{\frac{k}{m}}$$

$$\boxed{\omega_1 \approx \omega_{TMD}}$$

$$x(t)$$

$$y(t)$$

$$\ddot{x}_g(t)$$

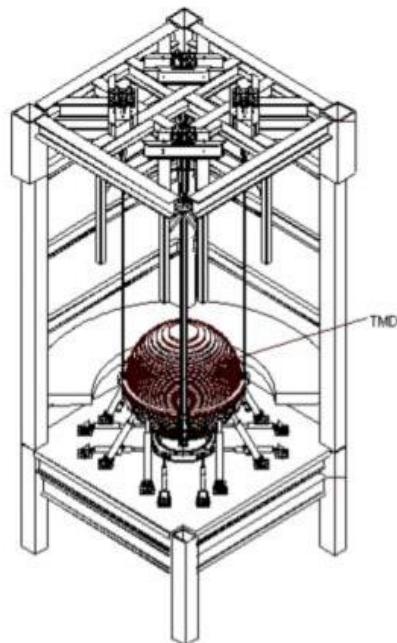
Ref.:  
 Adam C., Heuer R.,  
 Pirrotta A., 2003,  
 Experimental Mechanics,  
 Vol. 43(2), 124-130

$$\begin{cases} M\ddot{x} + (C + c)\dot{x} - c\dot{y} + (K + k)x - ky = -M\ddot{x}_g \\ m\ddot{y} - c\dot{x} + c\dot{y} - k\dot{x} + ky = -m\ddot{x}_g \end{cases}$$

# Passive control systems

## ● Tuned Mass Damper (TMD)

*Taipei 101 (Taiwan):*  
Height: 448 m; TMD mass: 660 t



# Passive control systems

## ● Tuned Mass Damper (TMD)

**Hancock Tower (Boston):**  
Height: 277 m;  
two TMDs of 300 t



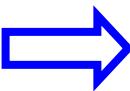
**Citicorp Building (New York):**  
Height: 278 m; TMD mass: 410 t



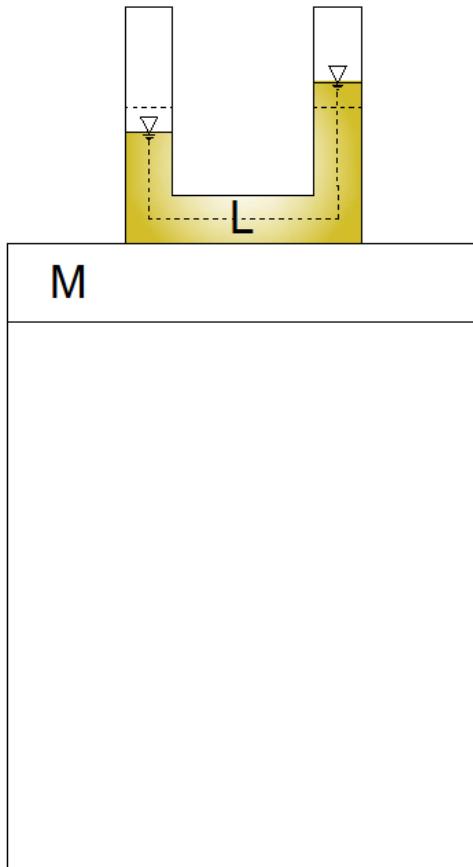
# ● Passive control systems

## ● Tuned Liquid Column Damper (TLCD)

Ref. F. Sakai, S. Takeda, T. Tamaki, 1989",  
Proc. of Intern. Conf. on highrise buildings, Nanjing, China



*Replaces the mass-spring-dashpot  
By a U-tube-like container where the  
motion of a liquid column absorbs part  
of vibration*



$$\omega_1 = \sqrt{\frac{K}{M}};$$

$$\omega_2 = \sqrt{\frac{2g}{L}};$$

g is the gravitational constant

- *Impart indirect damping effect to the primary structure through liquid oscillations*

- *The U-shaped tank is configured to have the natural frequency that optimally matches one or more of the structure's natural frequencies.*

$$\omega_1 \cong \omega_2$$

# Classical analytical formulation

Ref. T. Balendra, C.M. Wang, H.F. Cheong, 1995  
Engineering Structures, 17(9), 668-675

$$\begin{cases} (M + m_{TLCD})\ddot{x} + m_h \ddot{y} + C\dot{x} + Kx = -(M + m_{TLCD})\ddot{x}_g \\ m_h \ddot{x} + m_{TLCD} \ddot{y} + \frac{1}{2} \rho A \xi |\dot{y}| \dot{y} + 2\rho A g y = -m_h \ddot{x}_g \end{cases}$$

$M, C, K$  Mass, Damping and Stiffness  
of the main system

$$m_{TLCD} = \rho A (L_h + 2L_v) = \rho A L$$

$$m_h = \rho A L_h$$

$x$  Frame displacements

$y$  Vertical liquid displacement

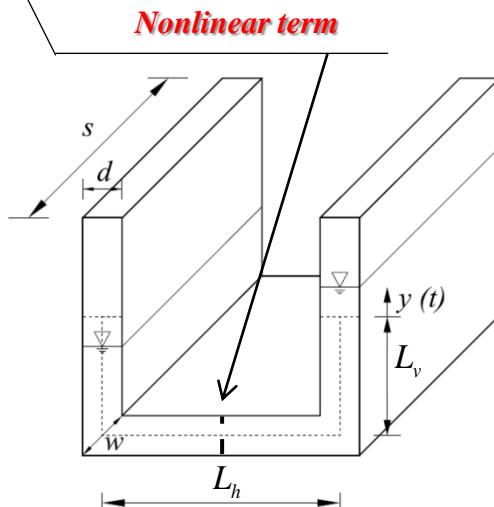
$\rho$  Density of the liquid inside the TLCD

$\xi$  Head loss coefficient

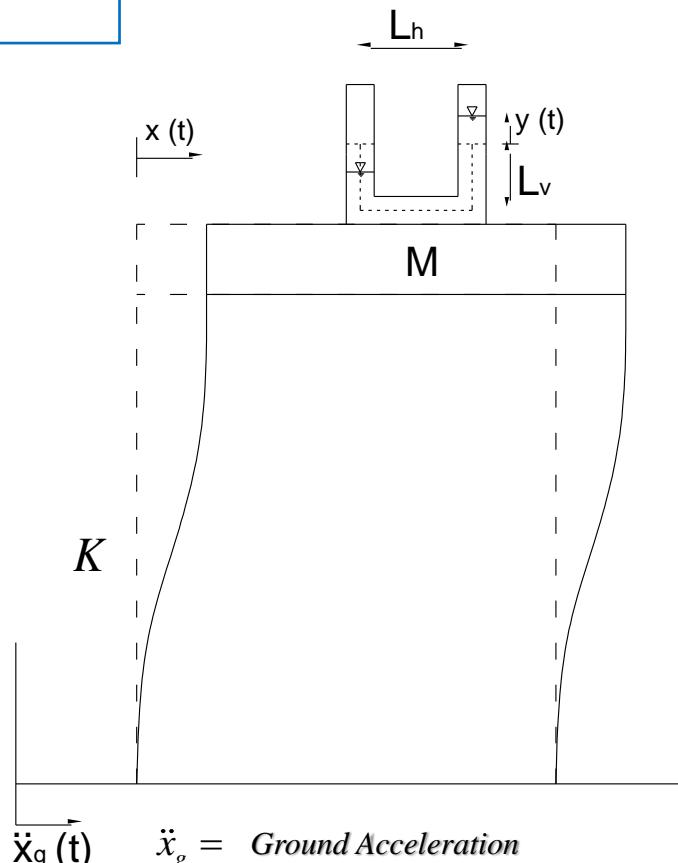
$g$  Gravity acceleration

$A$  Tube cross section

Tuning:  $\omega_1 \approx \omega_2$        $\omega_1 = \sqrt{\frac{K}{M}};$        $\omega_2 = \sqrt{\frac{2g}{L_h + 2L_v}};$



Ref.:  
Hochrainer M. J., Ziegler F., 2006,  
Control of tall building vibrations by  
sealed tuned liquid column dampers,  
Structural Control and Health  
Monitoring, Vol. 13, 980–1002

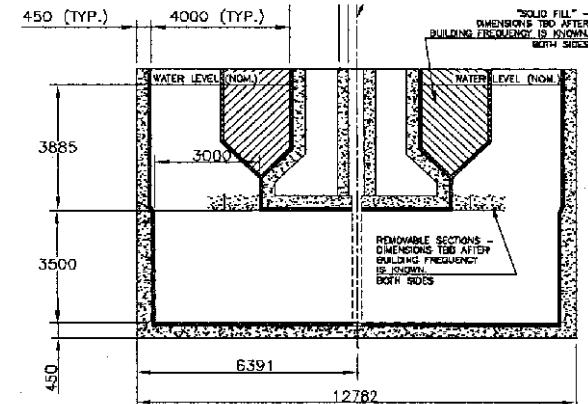
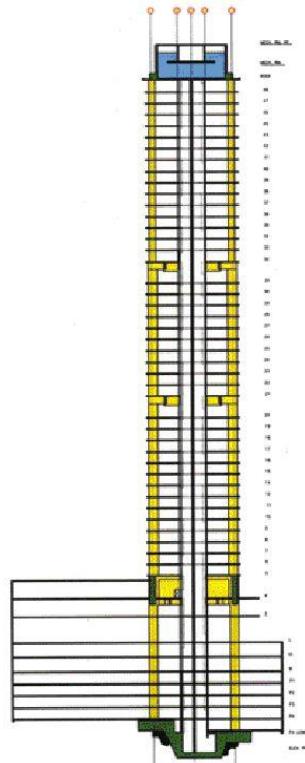


$\ddot{x}_g(t)$      $\ddot{x}_g =$  Ground Acceleration

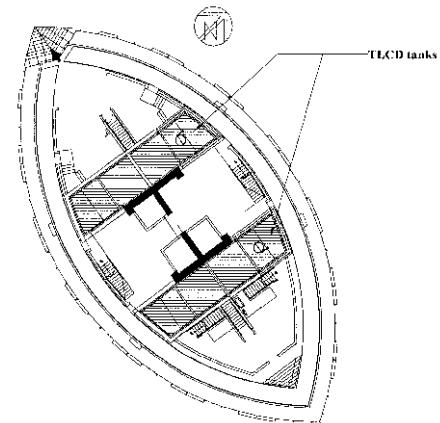
# Passive control systems

## Tuned Liquid Column Damper (TLCD)

**One Wall Center (Vancouver):**  
Height: 149.8 m; two TLCDs of 230 t



Tuned Liquid Column Damper of the One Wall Center

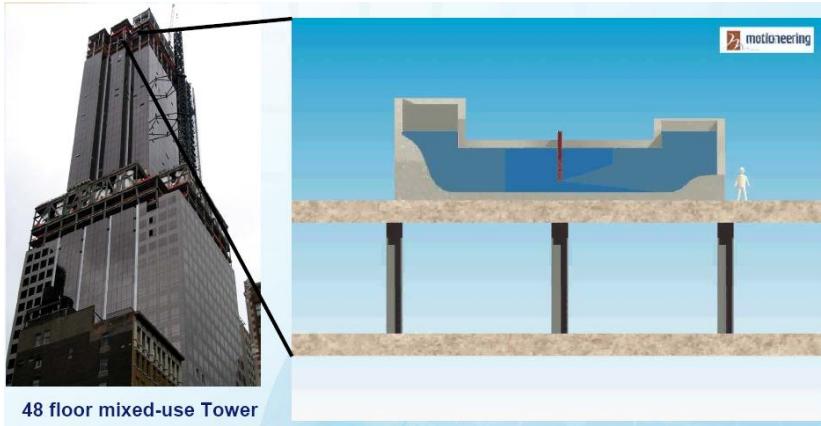


# Passive control systems

## Tuned Liquid Column Damper (TLCD)

**Random House (New York):**

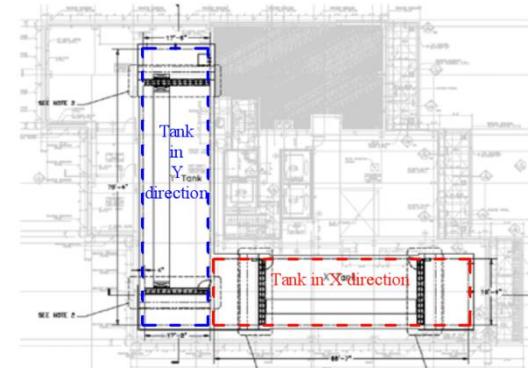
Height: 208 m; two TLCDs of 430 and 290 t



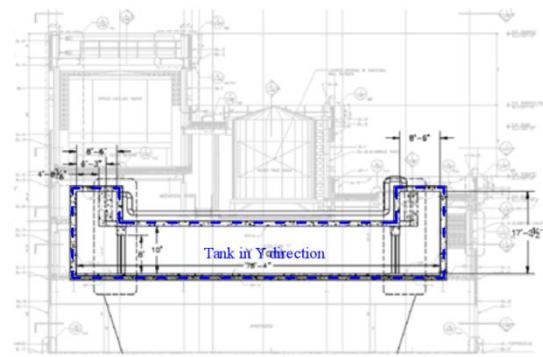
48 floor mixed-use Tower



East-West view of the Random House last floor



Random House 50° floor



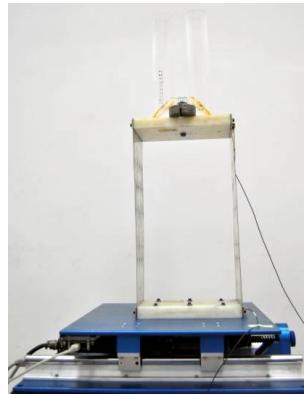
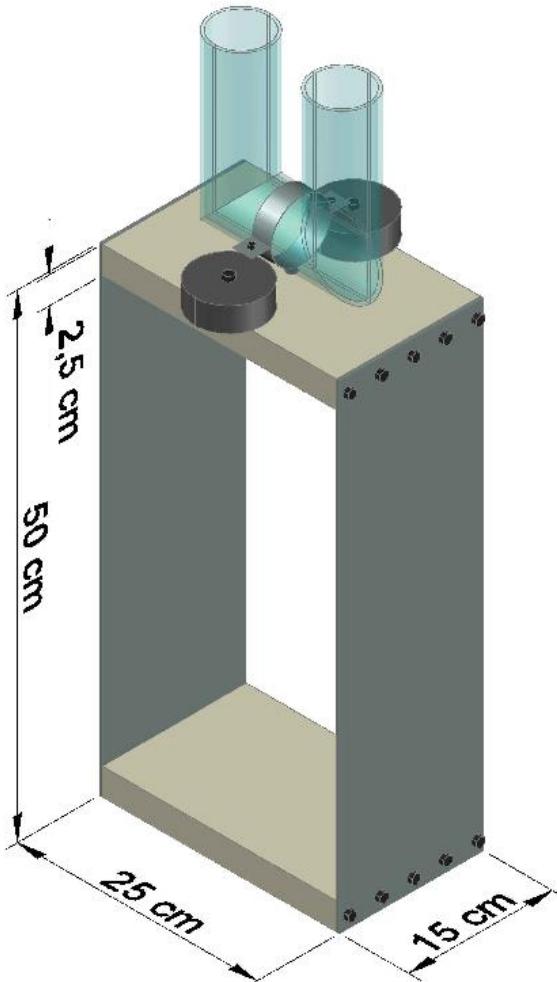
North-South view of the Random House last floor

 Effectiveness of the control

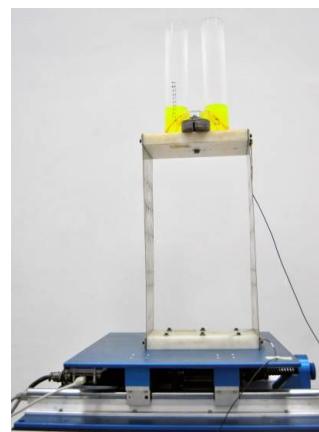
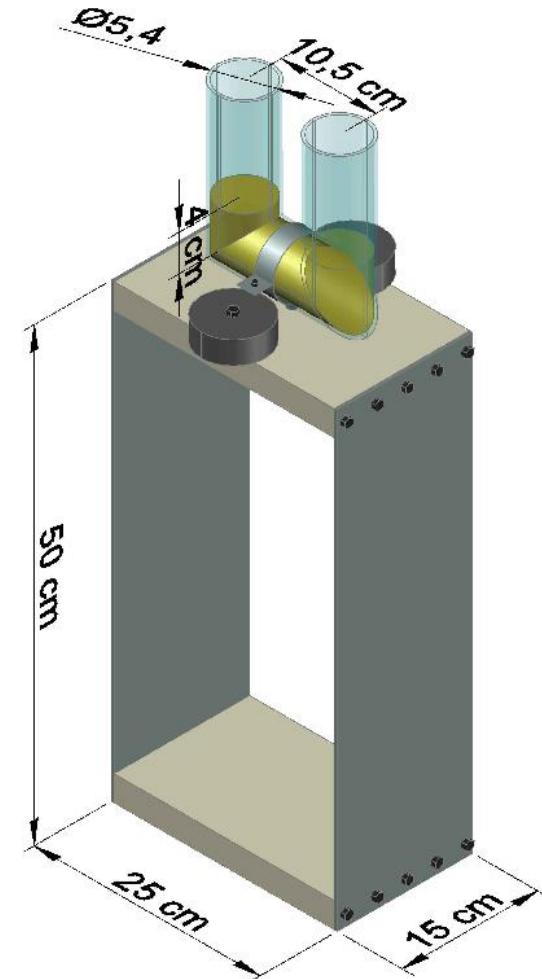
# EFFECTIVENESS OF THE CONTROL

Experimental dynamic Lab of University of Palermo

- Uncontrolled system: main system  
(no liquid inside)



- TLCD controlled system



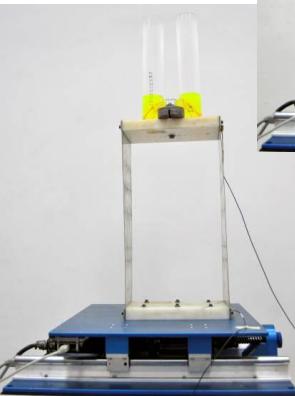
## Experimental tests



Effectiveness of the control

# Experimental validation

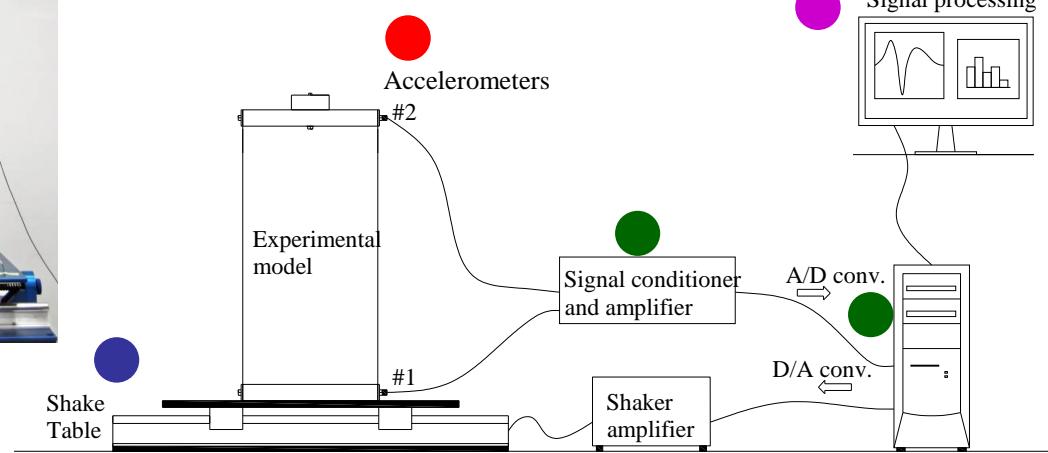
## ● Experimental setup



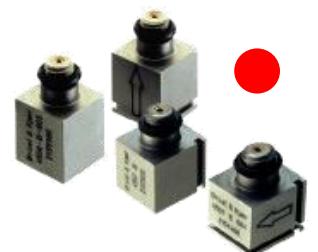
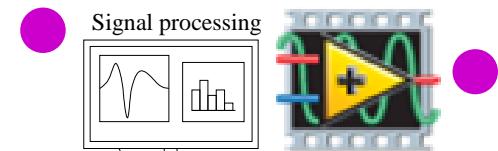
Quanser ShakeTableII  
1DOF shaking table  
45.5x45.5 cm



Experimental dynamic Lab of University of Palermo



NI Labview  
Virtual Instruments  
Matlab code



Brüel&Kjær piezoelectric  
accelerometer  
mod. 4507-002B  
(1000mV/g sensitivity)

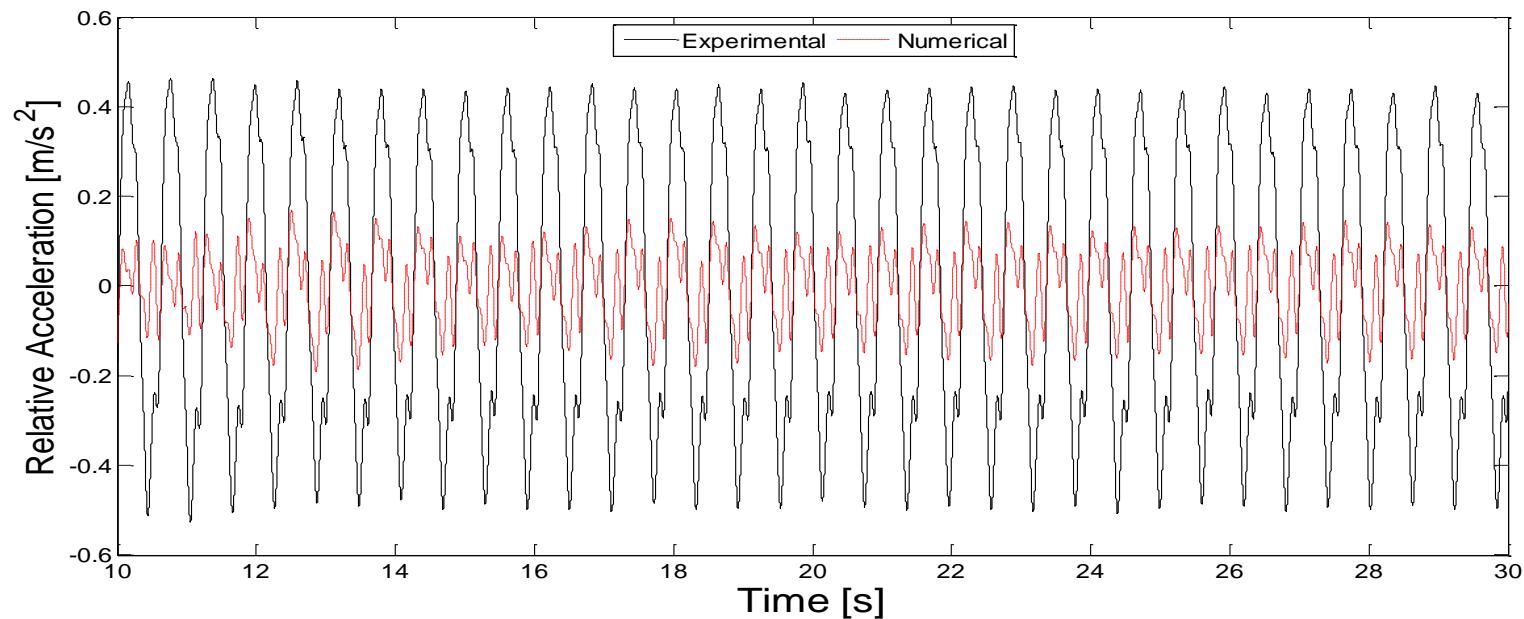
National Instruments PXIe-1082 DAQ system



# Numerical Vs Experimental results

$$\begin{cases} (M + m_{LCD})\ddot{x} + m_h \ddot{y} + C\dot{x} + Kx = -(M + m_{LCD})\ddot{x}_g \\ m_h \ddot{x} + m_{LCD} \ddot{y} + \frac{1}{2} \rho A \xi |\dot{y}| \dot{y} + 2\rho A g y = -m_h \ddot{x}_g \end{cases}$$

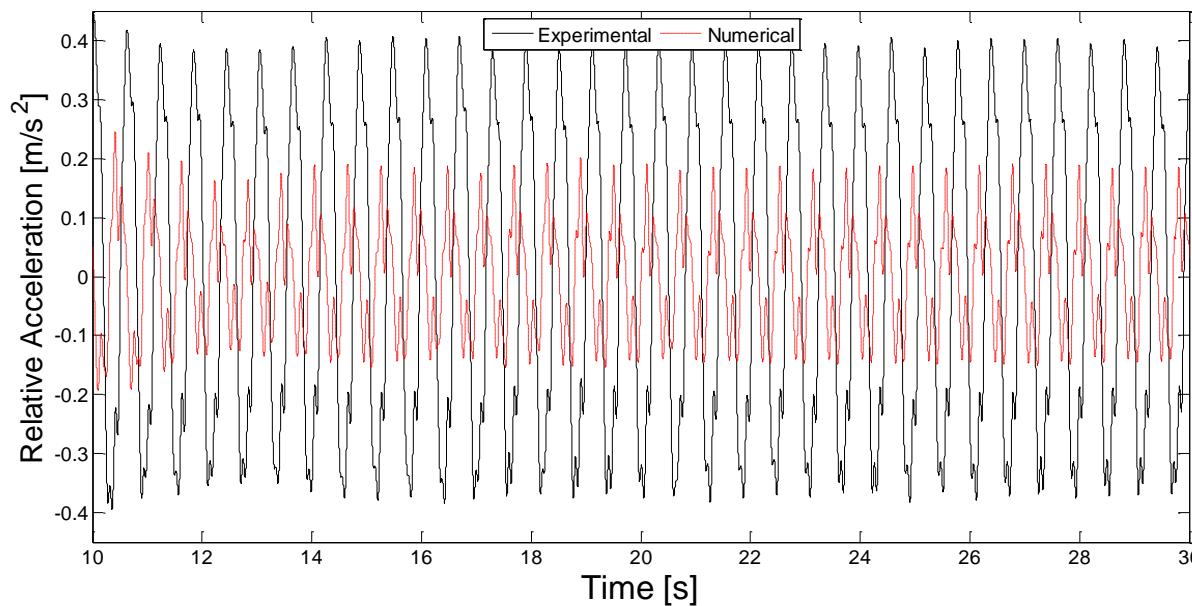
	Main Structures (uncontrolled system)	Controlled system
$M [kg]$	4.45027	4.45027
$C [Ns/m]$	0.7	0.7
$K [N/m]$	489.71	489.71
$\xi$		2
$A [cm^2]$		22.9
$L_h [cm]$		10.5



Sinusoidal test:  $f=1.65\text{Hz}$   
(resonance of the uncontrolled system)

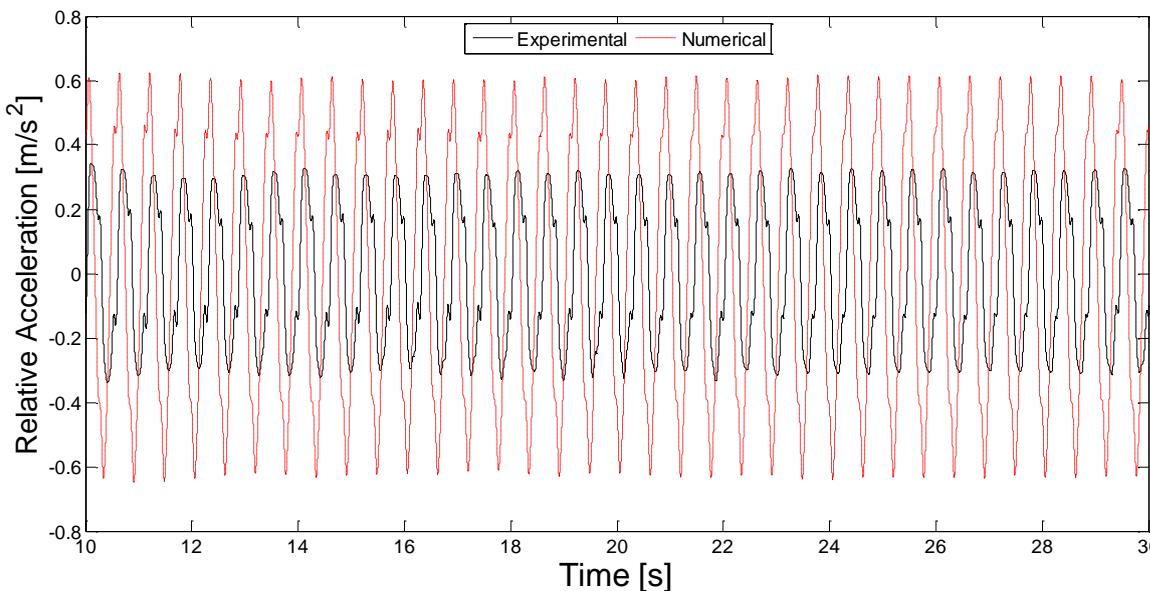
$L_v = 4 \text{ cm}$   
 $f_L = 1.63\text{Hz}$

# • Numerical Vs Experimental results



Sinusoidal test:  
 $f=1.65\text{Hz}$

$L_v = 5 \text{ cm}$



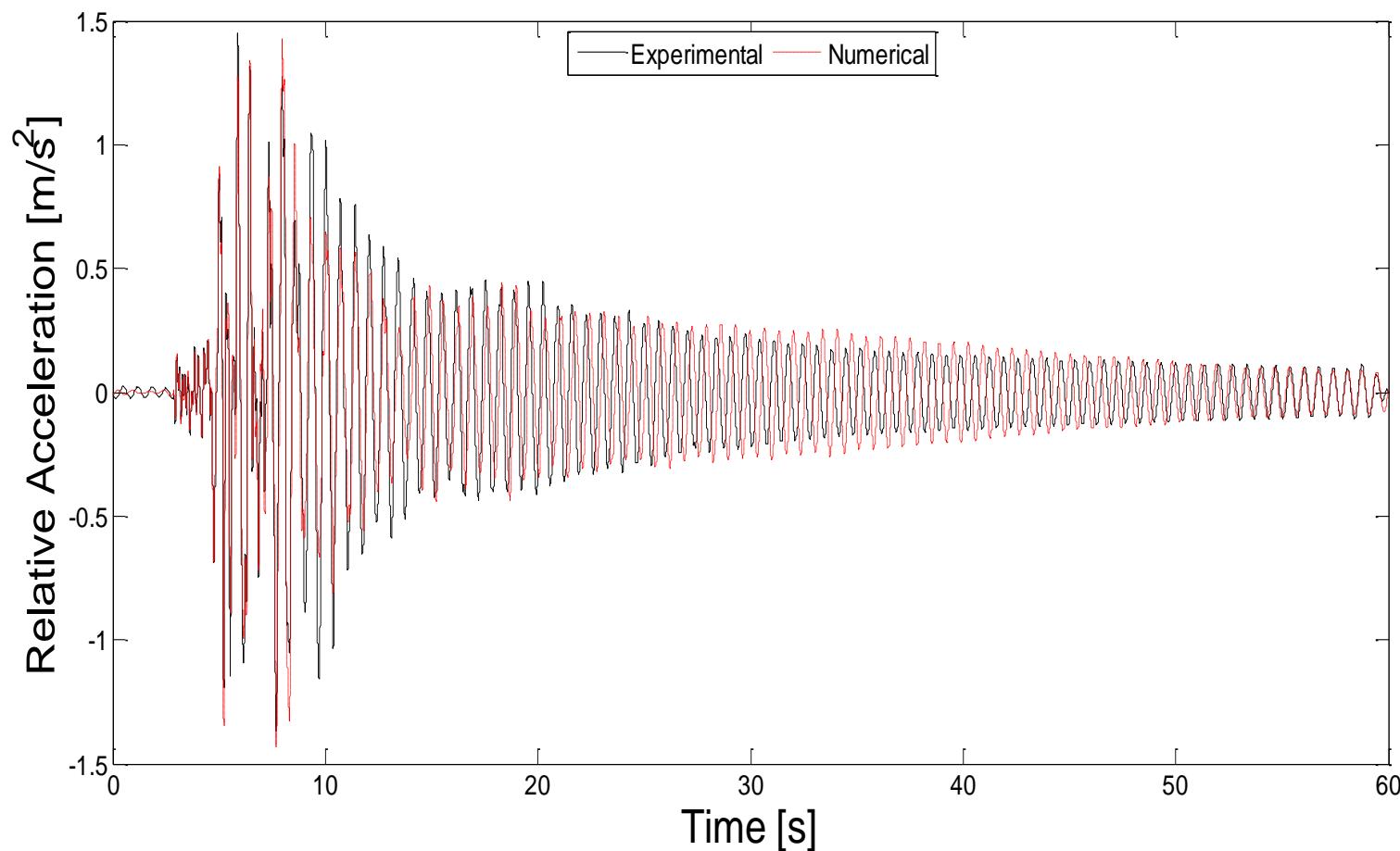
Sinusoidal test:  
 $f=1.75\text{Hz}$

$L_v = 5 \text{ cm}$

# • Numerical Vs Experimental results

**Tolmezzo acceleration**

$L_v = 4 \text{ cm}$



# VIRTUAL EXPERIMENT



Efficient Experiments allow to build up virtual experiments

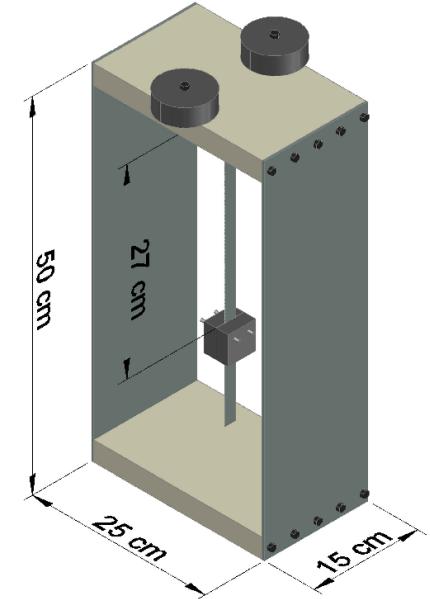
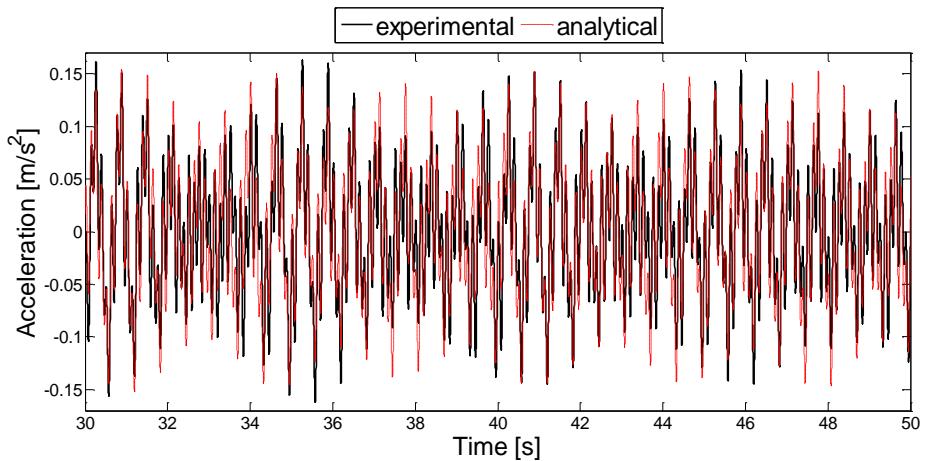
Once a proposed theoretical formulation is reliable  
having confirmed efficient tests

Simulations from this Mathematical formulation  
are  
virtual experiments

# Experimental validation

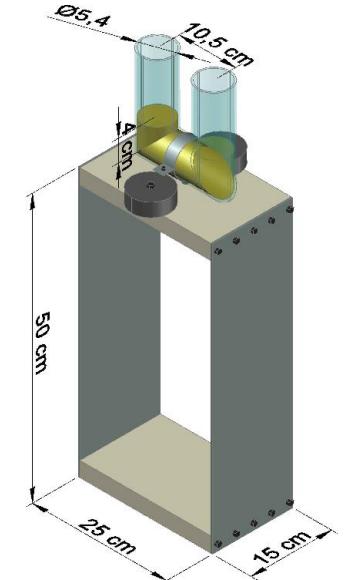
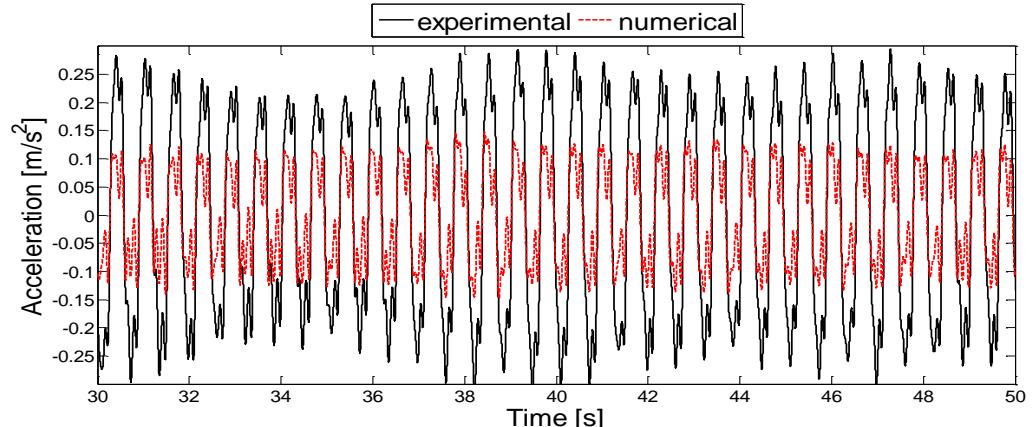
Sinusoidal test:  $f=1.60$  Hz

## Numerical-experimental comparison

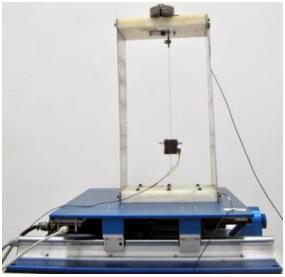


TMD- Virtual Experiment !!!

## TLCD- Virtual Experiment ???

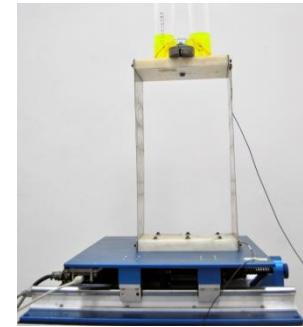


# controlled systems: Numerical-Experimental comparison

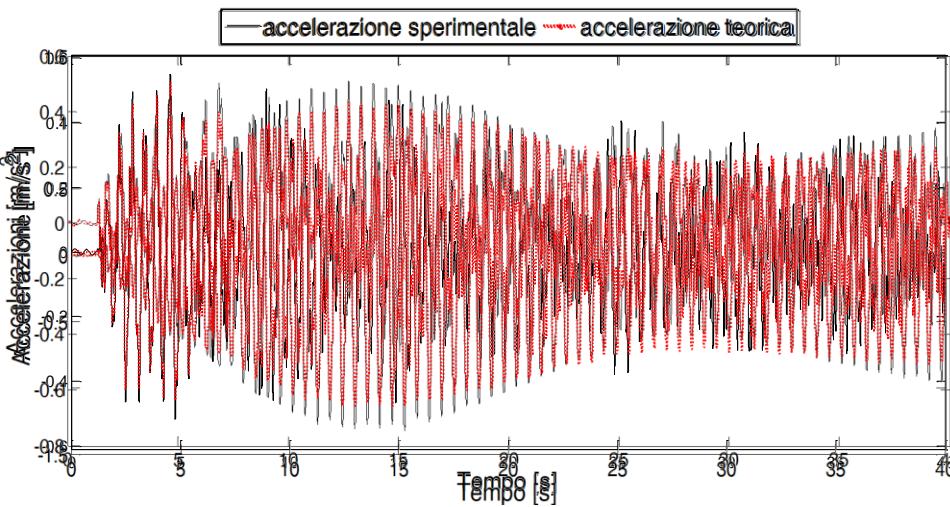


$$\ddot{x}_g = -\Omega^2 x_0 \sin(\Omega t)$$

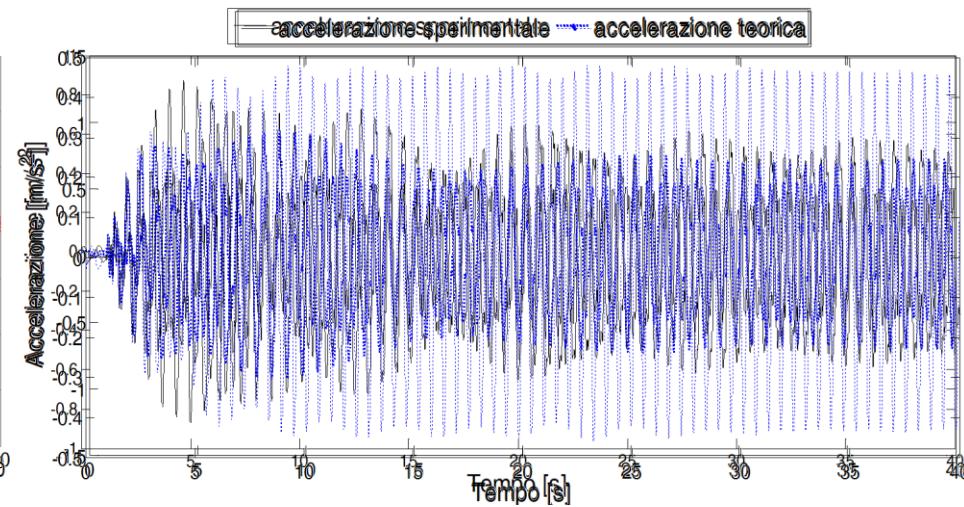
$$\Omega = 1,65 \text{ Hz} \quad x_0 = 0,5 \text{ mm}$$



**Acceleration response TMD controlled system**  
**Black experimental results**  
**Coloured theoretical results**

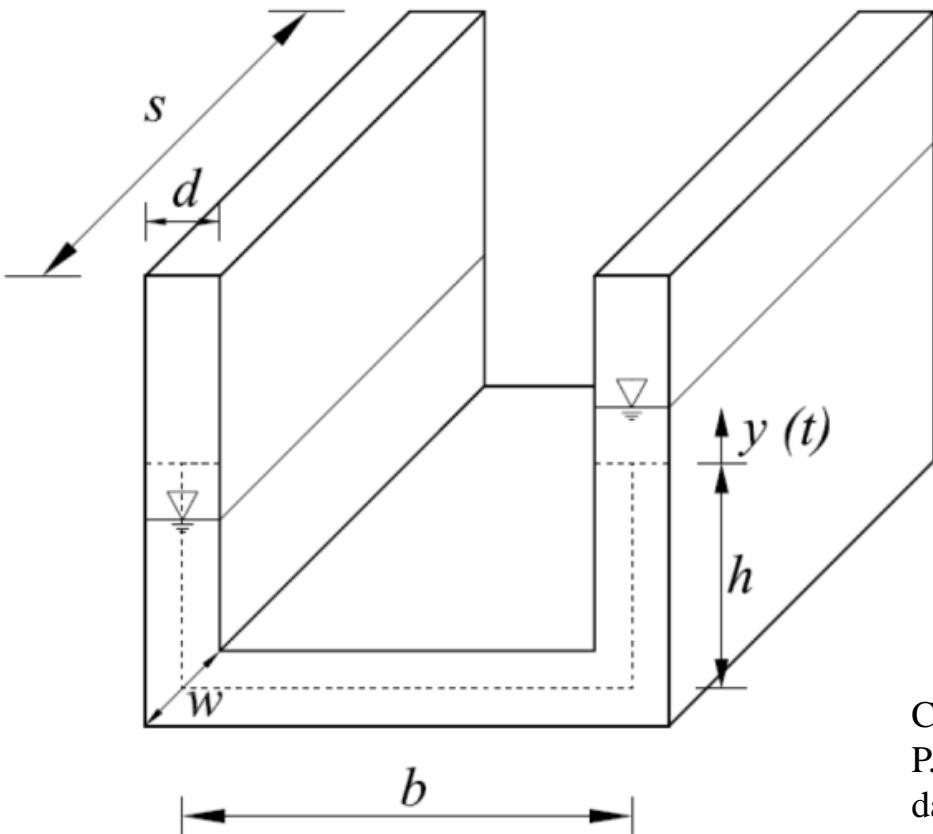


**Acceleration response TLCD controlled system**  
**Black experimental results**  
**Coloured theoretical results**



# WARNING

● Experimental validation  
of the classical analytical formulation  
of a Tuned Liquid Column Damper (TLCD) controlled system



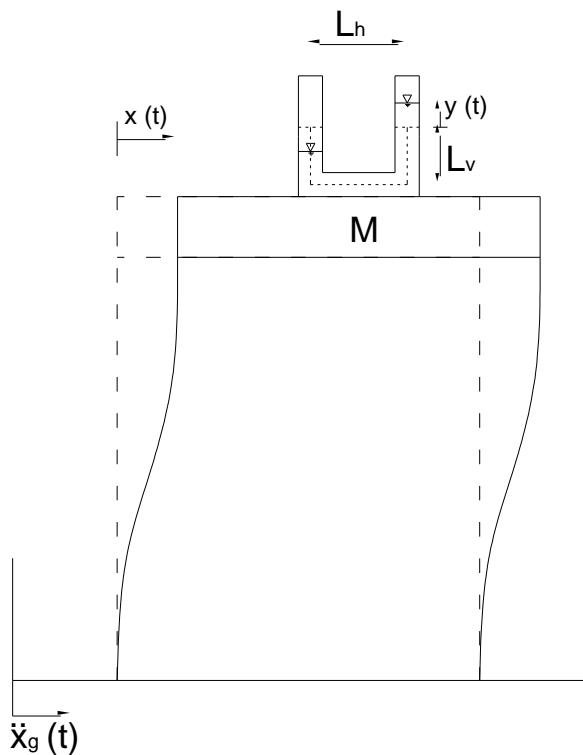
$w/b$

For a small ratio the classical nonlinear mathematical model is proper to predict the surface liquid motion.

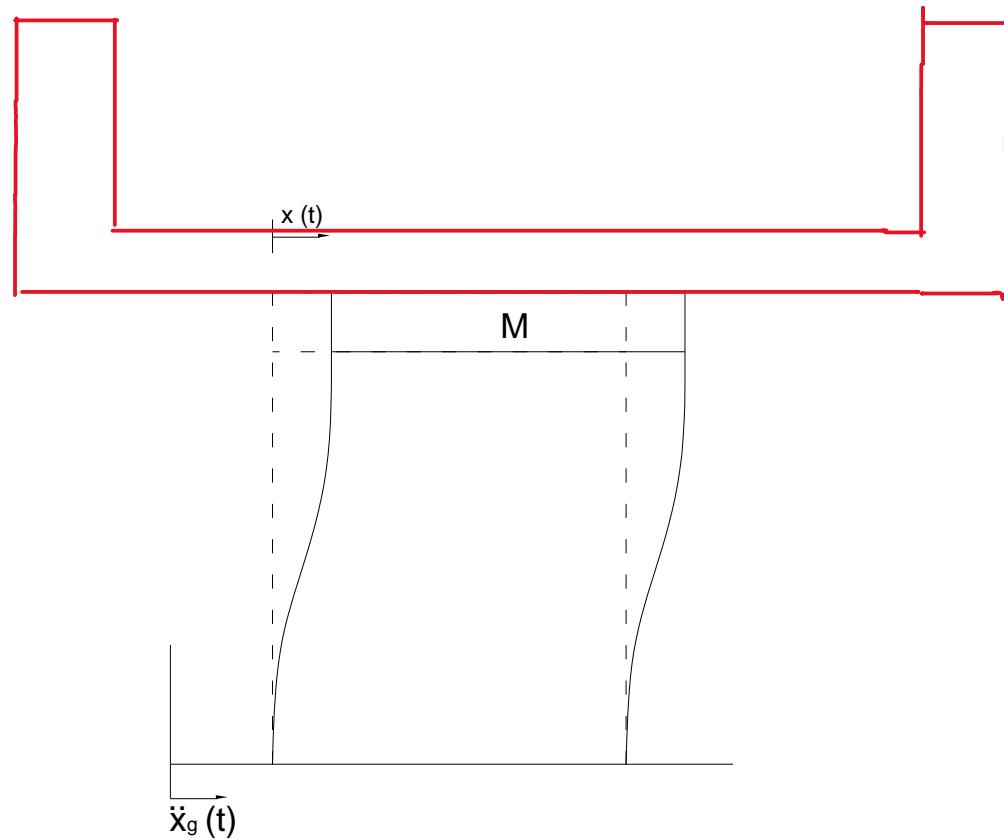
Chaiviriyawong P, Webster WC, Pinkaew T, Lukkunaprasit P. Simulation of characteristics of tuned liquid column damper using a potential-flow method. Eng Struct 2007;29:132–44.

# Experimental validation of the classical analytical formulation of a Tuned Liquid Column Damper (TLCD) controlled system

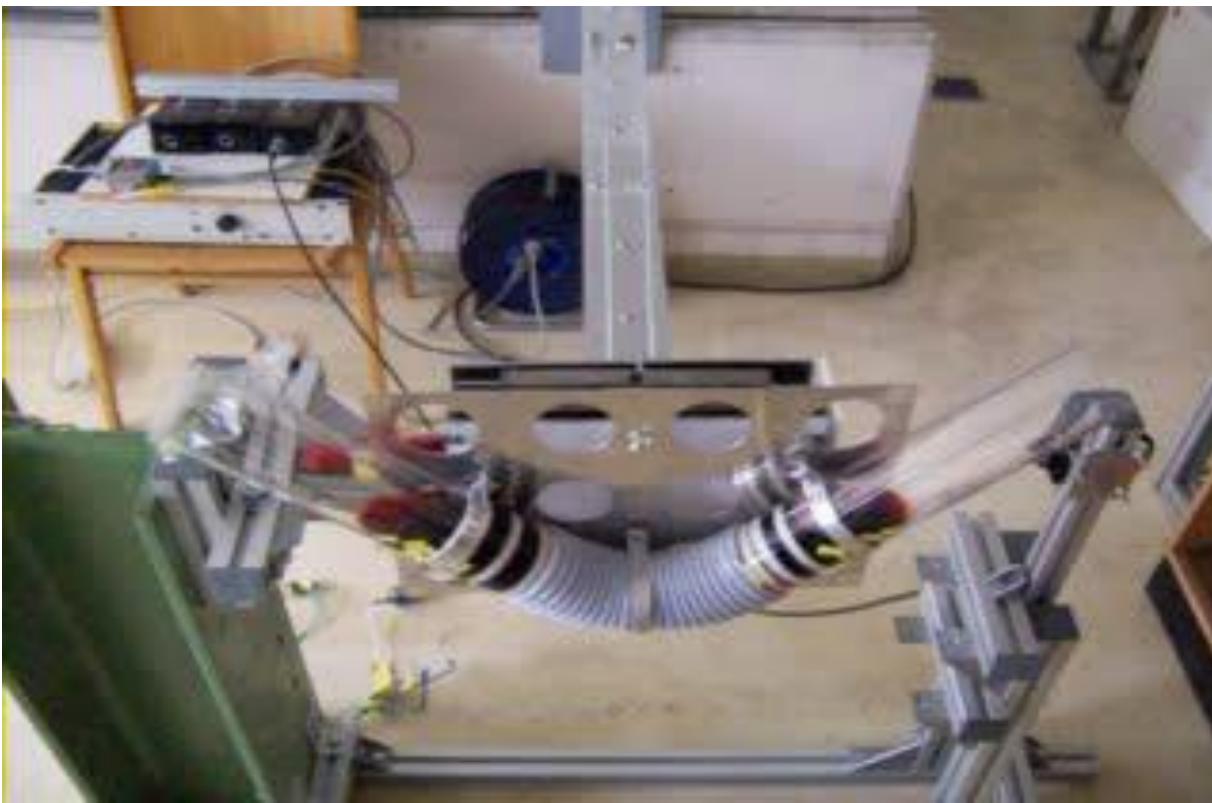
$w / b$



Small ratio is not generally appealing for structures since space constraints often limit the TLCD horizontal length



# H O W E V



University of Vienna

- ## E
- *TLCDs are relatively easy to install in new and existing buildings.*

## R

  - *It is easy to adjust their frequencies and they can be combined with active control mechanism (gas-spring effect).*

## I

  - *The water in the tank can be used for fire fighting (no need to add mass to the structure).*

## !

  - *Installation as well as maintenance cost is comparatively low.*

# A step Backward: the device only

The classical analytical formulation

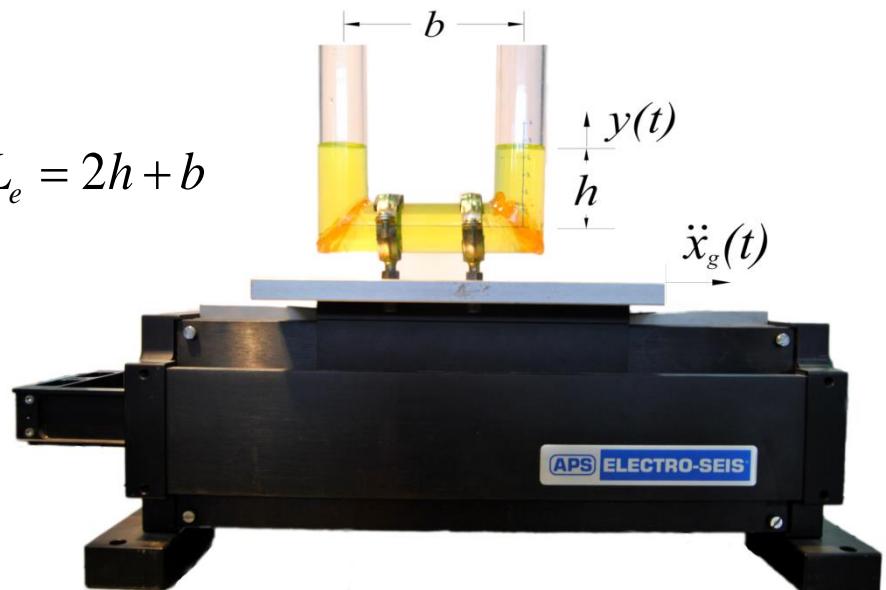
$$m_{TLCD} \ddot{y} + \frac{1}{2} \rho A \xi |\dot{y}| \dot{y} + 2\rho A g y = -m_h \ddot{x}_g$$

$$m_{TLCD} = \rho A (b + 2h) = \rho A L_e$$

$$m_h = \rho A b$$

$$\ddot{y}(t) + \frac{1}{2} \frac{\xi}{L_e} |\dot{y}(t)| \dot{y}(t) + \omega_0^2 y(t) = -\frac{b}{L_e} \ddot{x}_g(t)$$

$$\omega_0 = \sqrt{\frac{2g}{L_e}}$$



$y$  displacement of the liquid in the vertical columns,

$\rho$  density of the liquid inside the TLCD,

$\xi$  coefficient of head loss

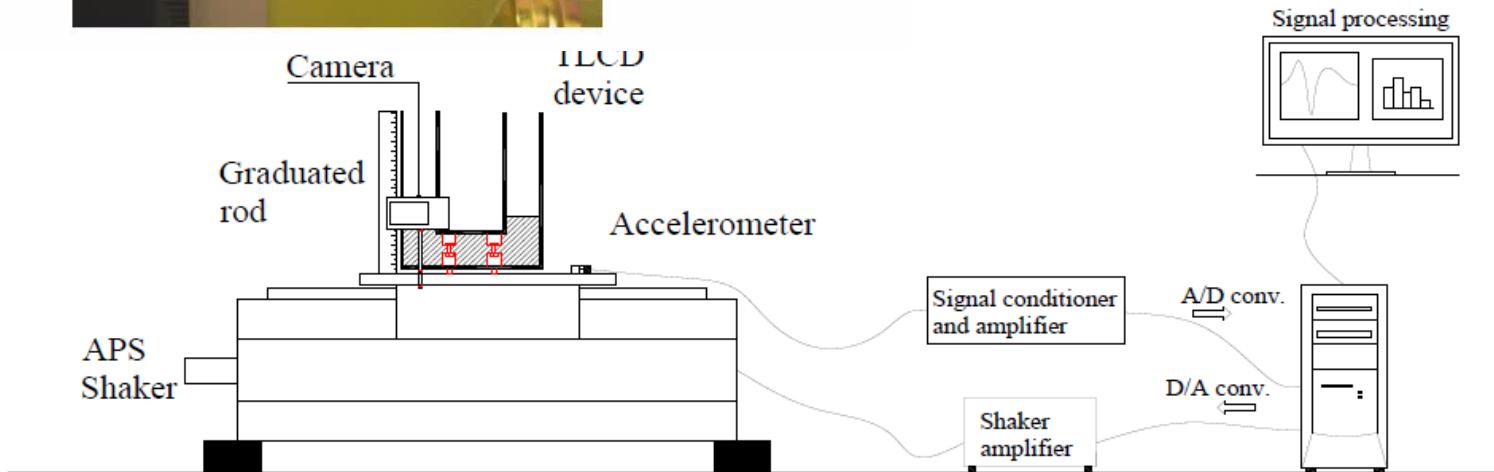
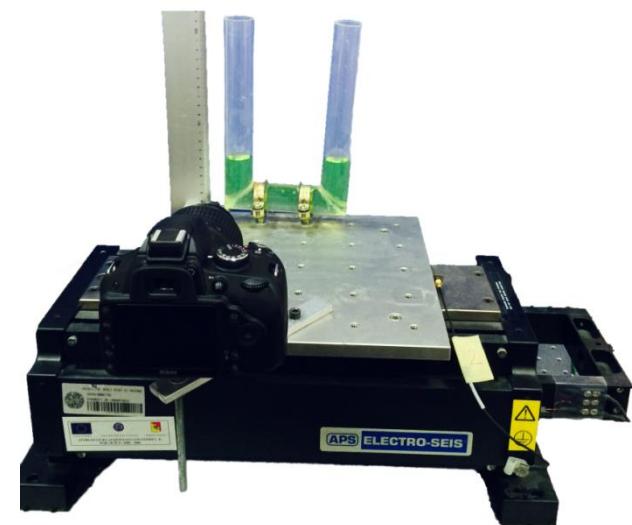
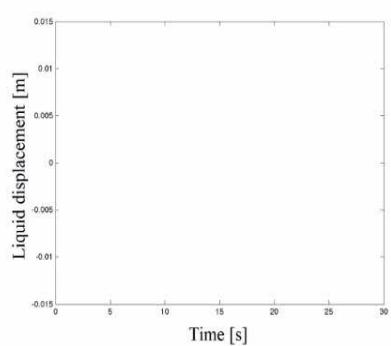
$g$  gravity acceleration

$A$  tube cross section

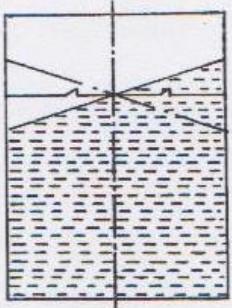
- H. Gao, K.C.S. Kwok, B. Samali, Optimization of tuned liquid column dampers, Eng. Struct., 19 (1997) 476-486.  
 C.C. Chang, C.T. Hsu, Control performance of liquid column vibration absorbers, Eng. Struct., 20 (1998) 580-586.



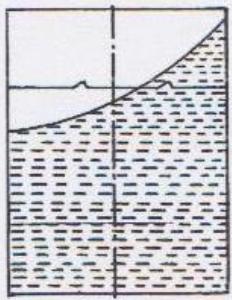
# Real dynamics



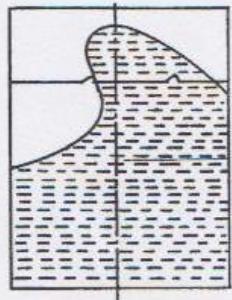
# First sloshing mode



(a) Linear modeling

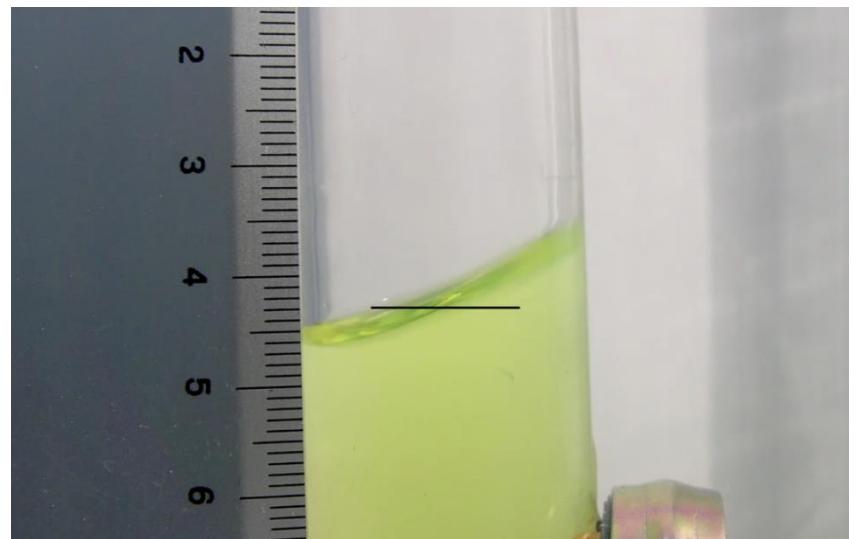
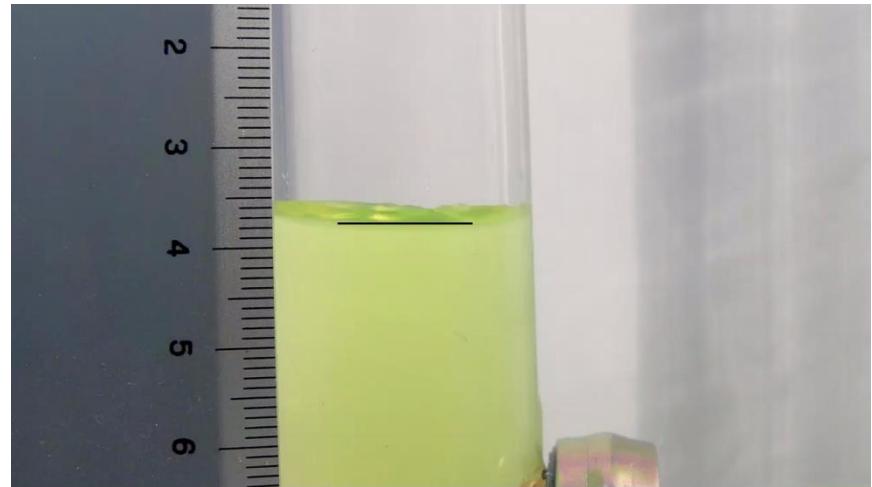


(b) Weakly nonlinear



(c) Sloshing impact

## Regimes of free-liquid-surface motion



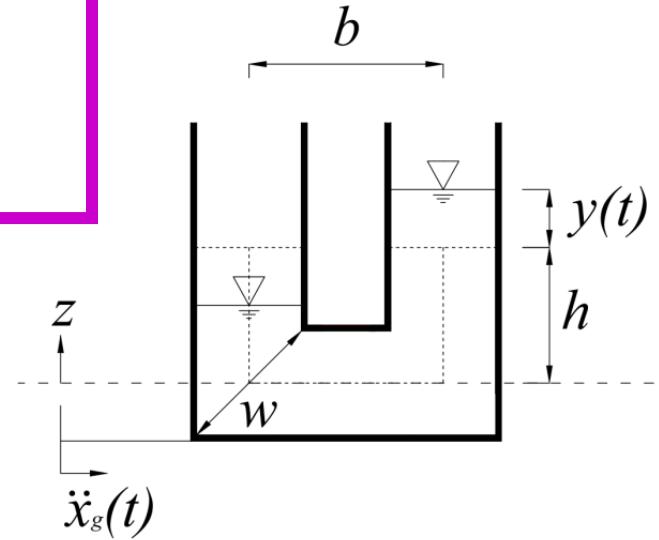
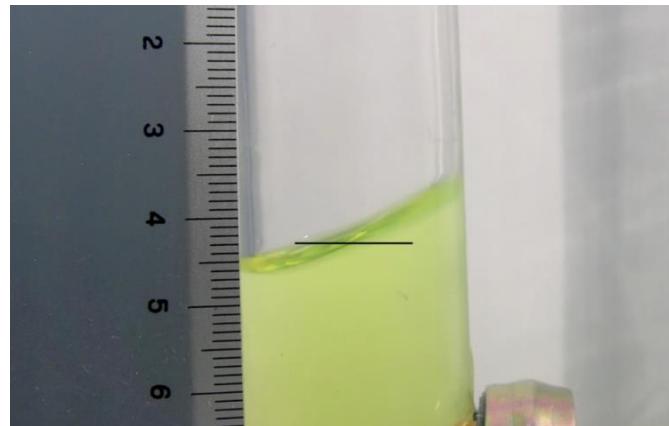
A first attempt to include the sloshing phenomenon, but without considering the simultaneous effect of sloshing and liquid vertical motion.

•T. Konar, A. Ghosh, Bimodal vibration control of seismically excited structures by the liquid column vibration absorbers, J. Vib. Control, (2012).

# SLOSHING INDUCED VARIATION OF THE FREQUENCY

$$\omega_0 = \sqrt{\frac{2g}{L_e}}$$

$$L_e = 2h + b$$

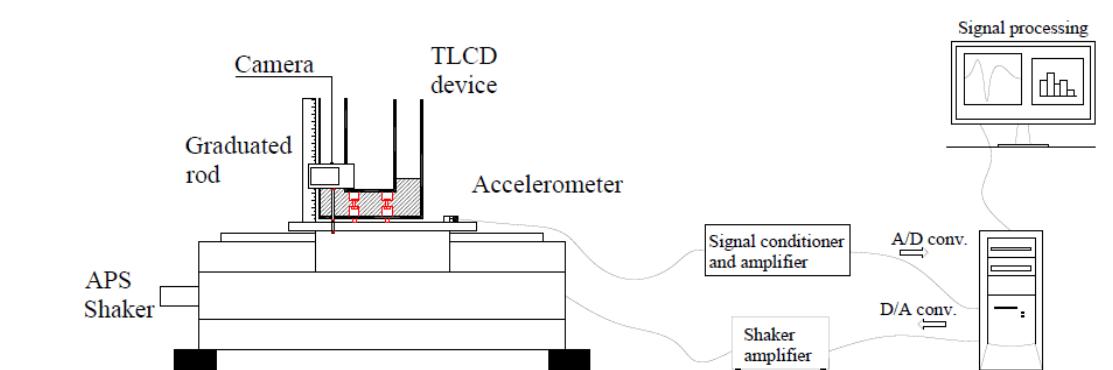
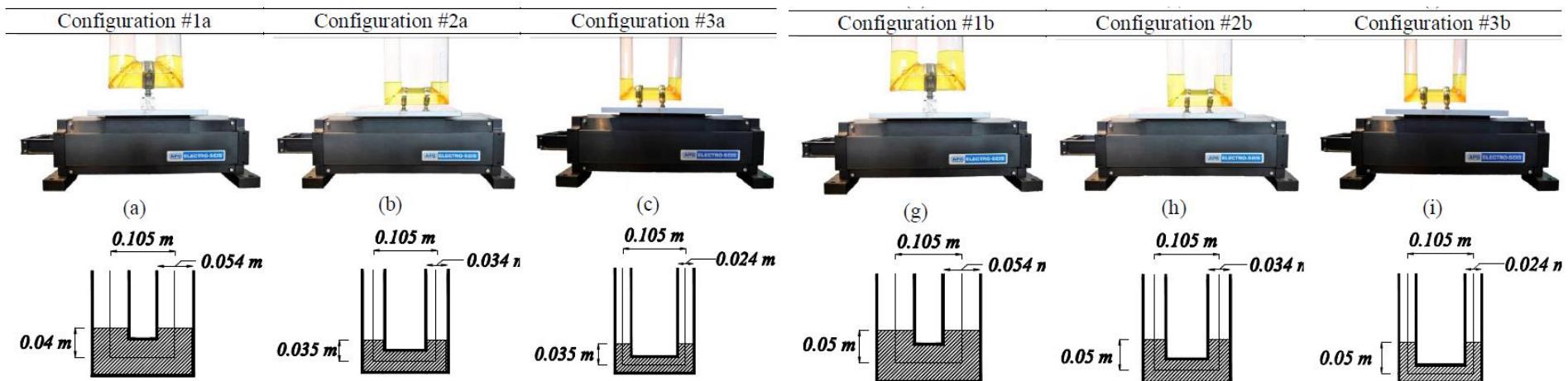


$$\tilde{\omega}_0 = \sqrt{\frac{2g}{\tilde{L}_e}}$$

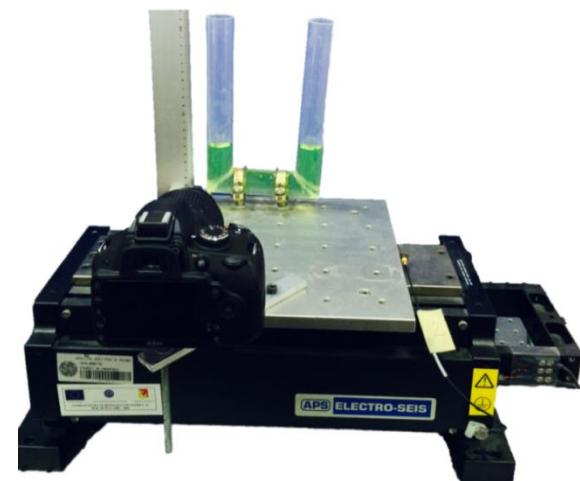
$$\tilde{L}_e = 2ph + b$$

Effective Length; p experimentally determined

# Experimental investigation on the TLCD device



**Experimental dynamic laboratory (DICAM),  
Palermo**





## *Innovative Modelling for TLCD Controlled Structures*

# Innovative Modelling for Capturing Sloshing in TLCD

Nonlinear

Classical  
equation of motion

Linear

Proposed linear fractional model

$$\ddot{y}(t) + \frac{1}{2} \frac{\xi}{L_e} |\dot{y}(t)| \dot{y}(t) + \omega_0^2 y(t) = -\frac{b}{L_e} \ddot{x}_g(t)$$

$$\omega_0 = \sqrt{\frac{2g}{L_e}}$$

$$\ddot{y}(t) + \frac{1}{2} \frac{C_\beta}{L_e} \left( {}_0^C D_t^\beta y(t) \right) + \omega_0^2 y(t) = -\frac{b}{L_e} \ddot{x}_g(t)$$

Caputo's Fractional  
Derivative

$${}_0^C D_t^\beta y(t) = \frac{1}{\Gamma(1-\beta)} \int_0^t (t-\tau)^{\beta} \frac{d}{d\tau} y(\tau) d\tau, \quad 0 < \beta < 1$$

$$\Im({}_0^C D_t^\beta y(t)) = (i\omega)^\beta \Im(y(t))$$

$$H(\omega) = \frac{-\frac{b}{L_e}}{-\omega^2 + \frac{1}{2} \frac{C_\beta}{L_e} (i\omega)^\beta + \omega_0^2}$$

Euler Gamma Function     $\Gamma(\beta) = \int_0^\infty e^{-z} z^{\beta-1} dz$      $0 < \beta < 1$

$$\beta - 1 ! = \Gamma \beta$$

# fractional calculus



Engineers

are

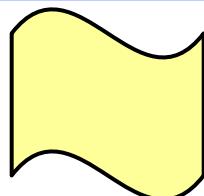
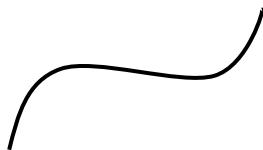
Euclidean

People

# Considering a mechanical model

## Engineer are Euclidean People

•



POINT

$[L^0]$

LINE

$[L^1]$

SURFACE

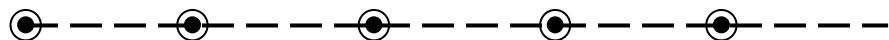
$[L^2]$



VOLUME

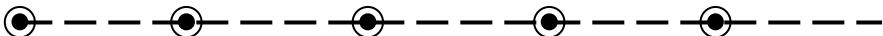
$[L^3]$

DIMENSIONS



0 VOID 1 VOID 2 VOID 3 VOID 4

DERIVATIVES  $d^j f / dx^j = (D^j f)(x)$



0 VOID 1 VOID 2 VOID 3 VOID 4

PRIMITIVES

$$\iiint f(x) \dots dx = (I^j f)(x)$$



0 VOID 1 VOID 2 VOID 3 VOID 4



# Fractional Calculus

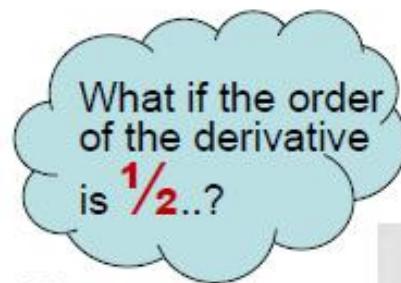
Fractional Calculus was born in 1695

De l'Hopital asked Leibniz

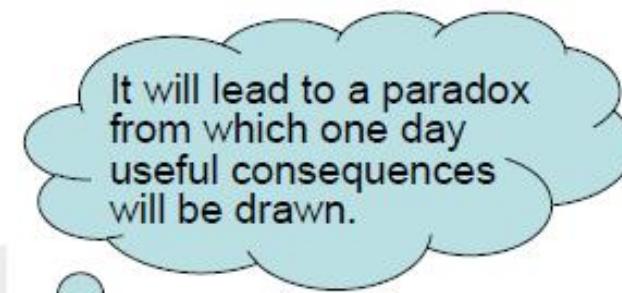
- Background



G. De l' Hopital  
(1661-1704)



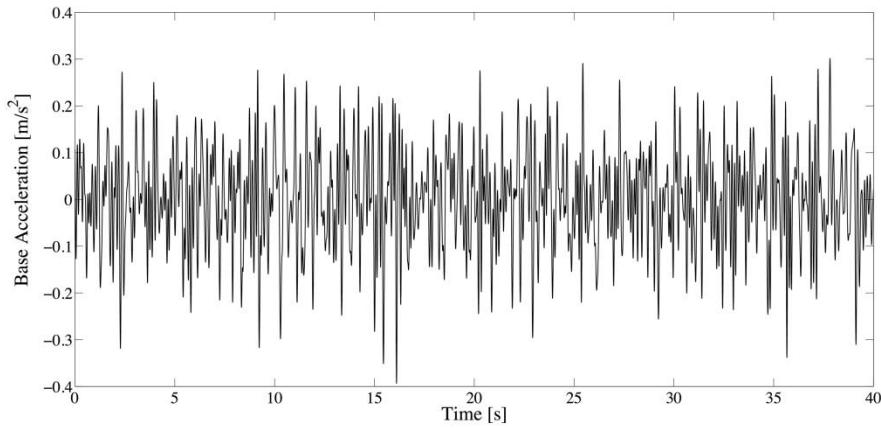
G. Leibniz  
(1646-1716)



## Fractional Calculus Diverse Impact

- Materials Science
- Theoretical Physics
- Financial Mathematics
- ...

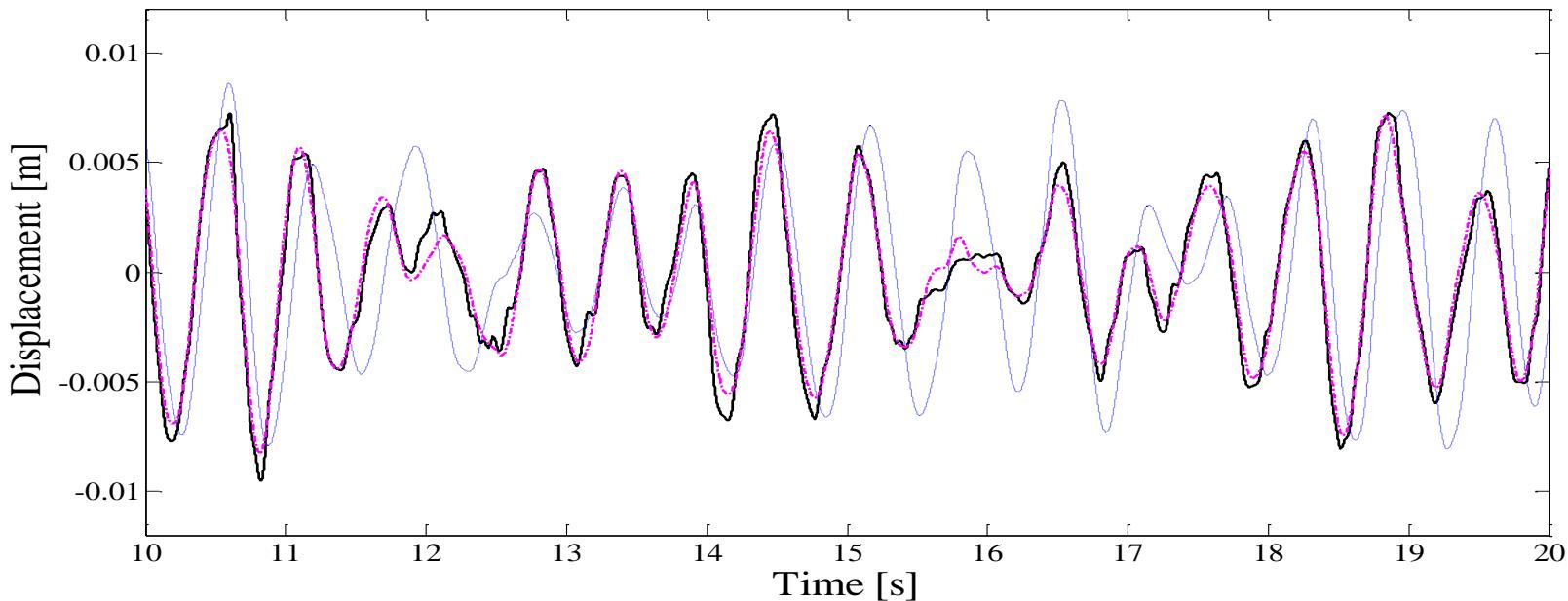
# Numerical Vs Experimental results



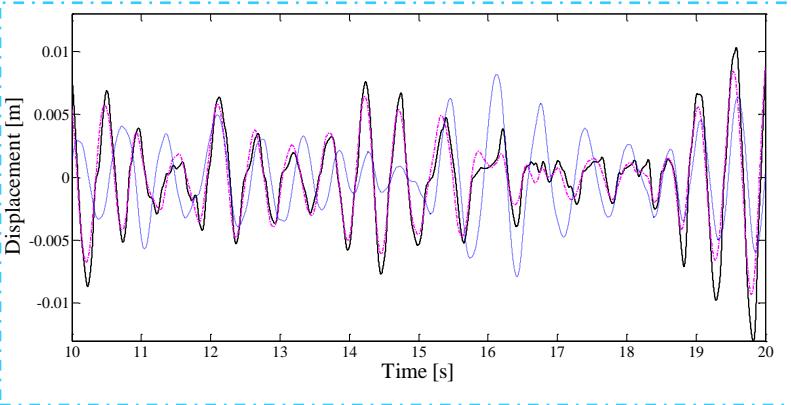
Broadband noise 0.5-10Hz



— experimental    - - - Classical numerical    - · - Proposed model



# The TLCD device: Numerical-experimental comparison



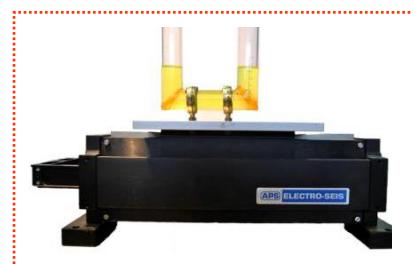
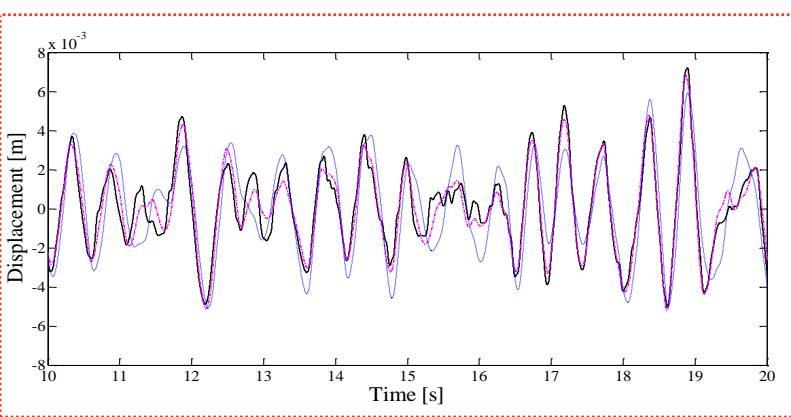
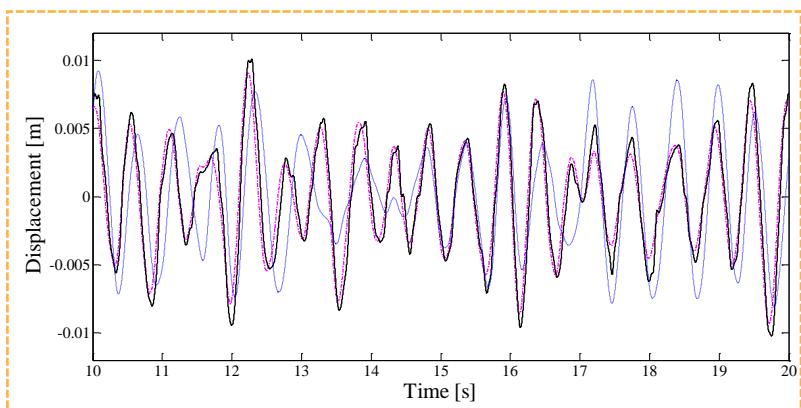
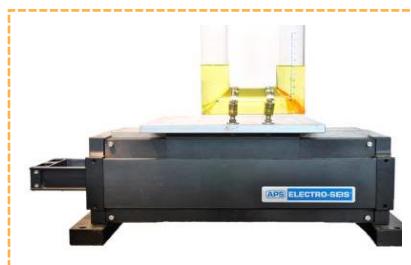
— Experimental  
- - - Classical numerical  
- - - - Proposed fractional model



$$\beta = 0.2$$

$$C_\beta = 7.4$$

Ref.: Di Matteo A., Lo Iacono F., Navarra G., Pirrotta A., 2015, Comm. in Nonlinear Science and Numerical Simulation, 23 (229-244)



Broadband noise 0.5-10Hz  
20 Samples of 25s

$$\beta = 0.6$$

$$C_\beta = 2.1$$

# TLCD controlled system: Proposed model with fractional derivative

*Classical equation of motion*

$$\begin{cases} (M + m_{TLCD})\ddot{x} + m_h\ddot{y} + C\dot{x} + Kx = -(M + m_{TLCD})\ddot{x}_g \\ m_h\ddot{x} + m_{TLCD}\ddot{y} + \frac{1}{2}\rho A\xi|\dot{y}|\dot{y} + 2\rho Agy = -m_h\ddot{x}_g \end{cases}$$



*Proposed linear fractional model*

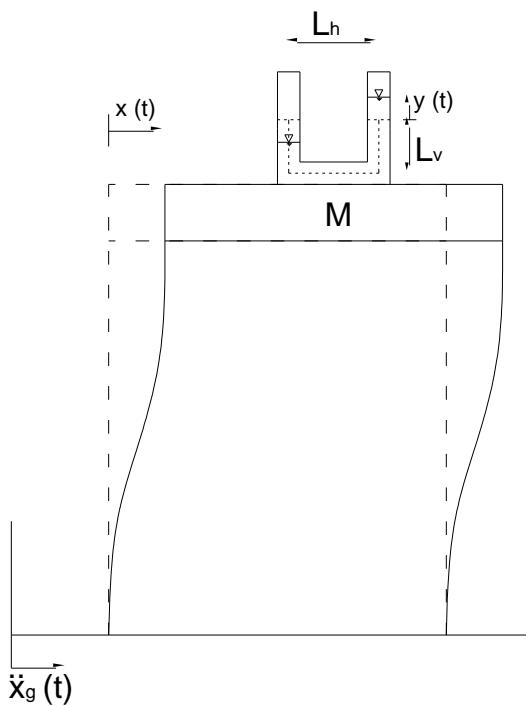
$$\begin{cases} (M + m_{TLCD})\ddot{x} + m_h\ddot{y} + C\dot{x} + Kx = -(M + m_{TLCD})\ddot{x}_g \\ m_h\ddot{x} + m_{TLCD}\ddot{y} + \frac{1}{2}\rho AC_\beta \left( {}_0D_t^\beta y(t) \right) + 2\rho Agy = -m_h\ddot{x}_g \end{cases}$$

$M, C, K$

*Main system parameters*

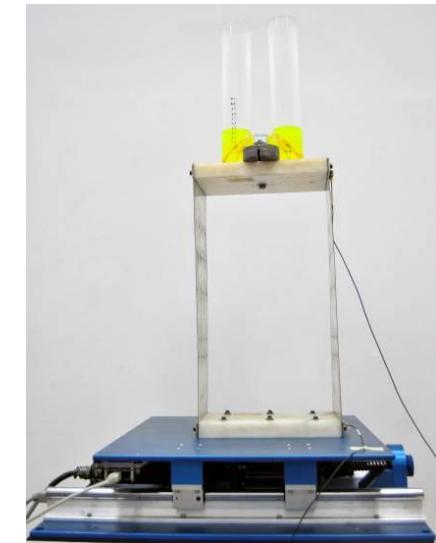
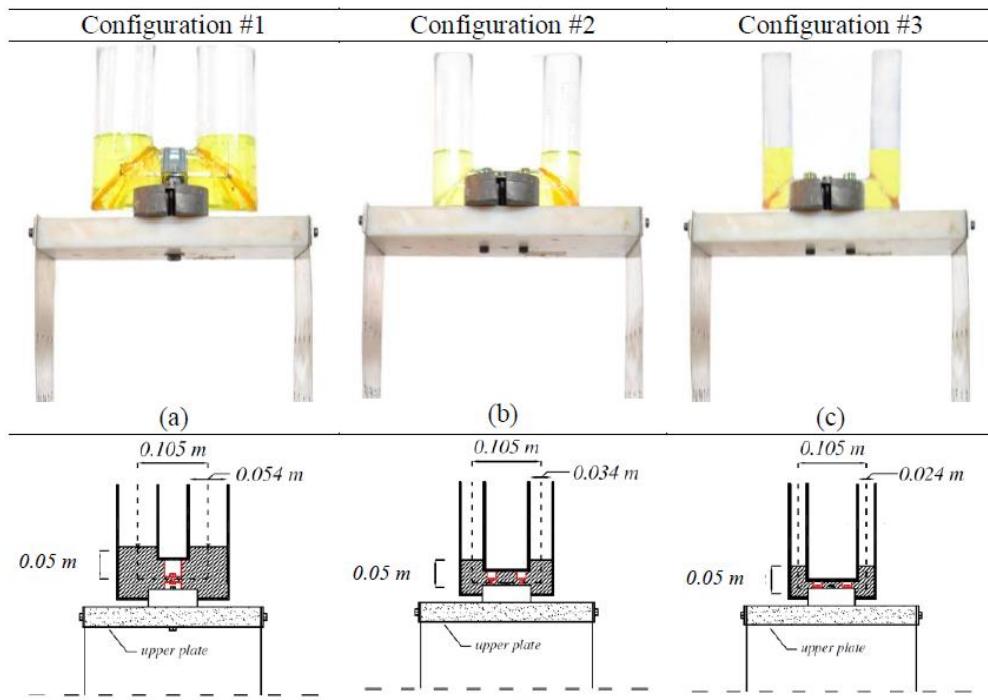
$$m_{TLCD} = \rho A(L_h + 2L_v) = \rho AL$$

$$m_h = \rho A L_h$$



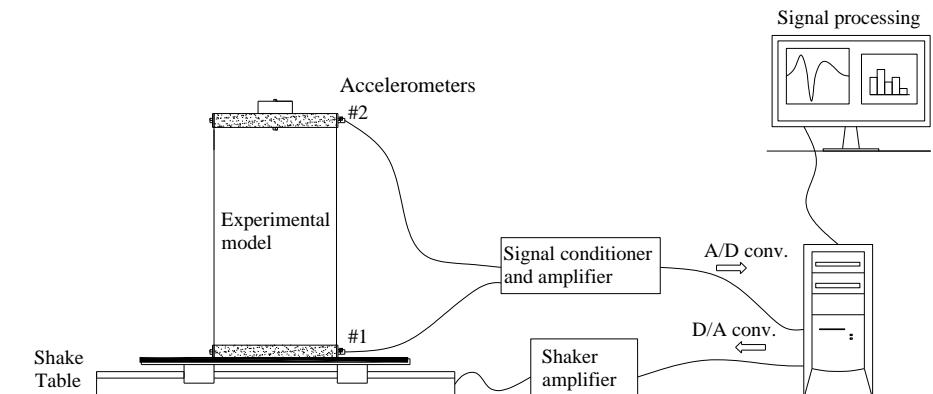
Ref.: Di Matteo A., Di Paola M., Pirrotta A.; Innovative modeling of Tuned Liquid Column Damper controlled structures, 2015, Smart Structures and Systems, Submitted.

# TLCD controlled system: Experimental validation of the proposed model

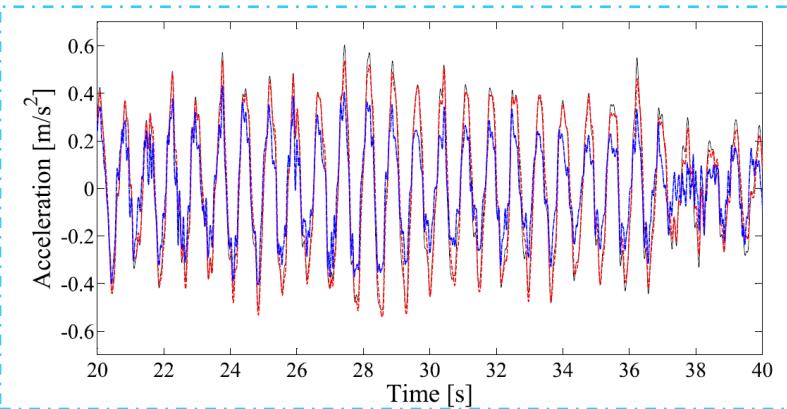


**Experimental dynamic laboratory (DICAM),  
Palermo**

	Configuration #1	Configuration #2	Configuration #3
$M$	4.486 kg	4.226 kg	4.208 kg
$\zeta_1$	0.005	0.0032	0.0037
$\omega_1$	9.10 rad/s	9.54 rad/s	9.65 rad/s
$L$	0.205 m	0.205 m	0.205 m
$\omega_2$	9.78 rad/s	9.78 rad/s	9.78 rad/s
$\omega_{\text{exp}}$	11.31 rad/s	10.81 rad/s	10.56 rad/s
$\xi$	6	8	15



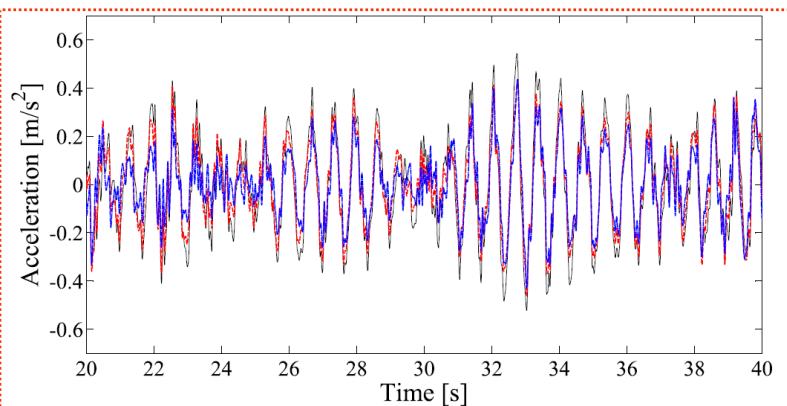
# TLCD controlled system: Numerical-Experimental comparison (time domain)



Broadband noise 0.5-10Hz  
10 Samples of 60s

$$\beta = 0.4 \\ C_\beta = 3.6$$

**Experimental dynamic laboratory (DICAM),  
Palermo**

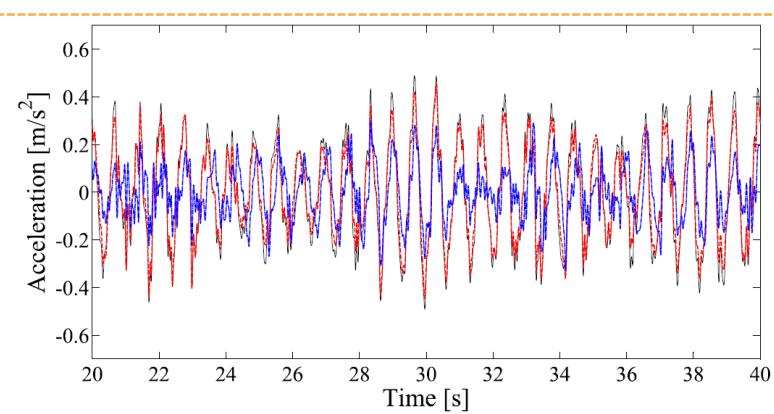


★  
 $\beta = 0.15$   
 $C_\beta = 9.9$

— Experimental

Classical  
numerical

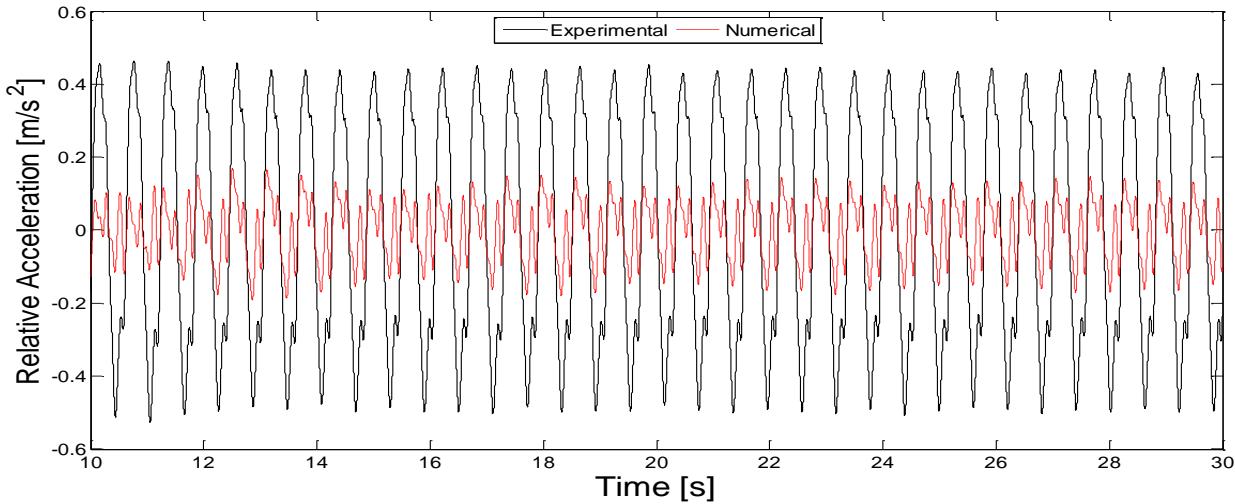
Proposed fractional  
model



★  
 $\beta = 0.6$   
 $C_\beta = 2.1$

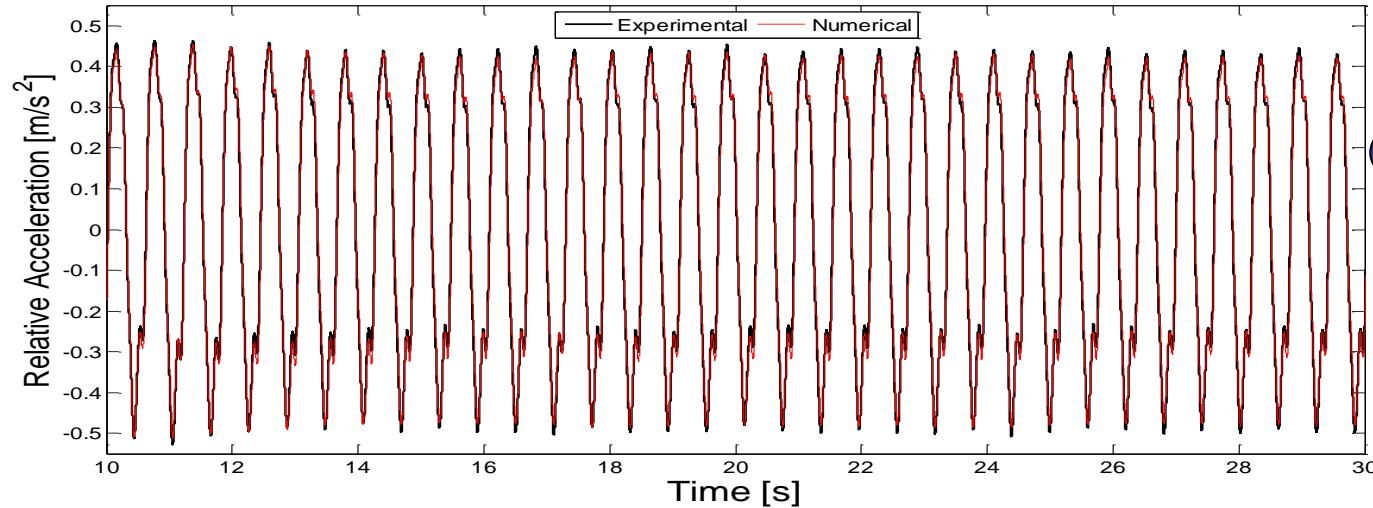
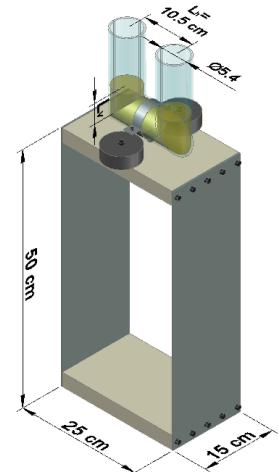
Ref.: Di Matteo A., Di Paola M., Pirrotta A.;  
Innovative modeling of Tuned Liquid Column  
Damper controlled structures, 2015, Smart  
Structures and Systems, Submitted.

# Numerical Vs Experimental results



$L_v = 4 \text{ cm};$

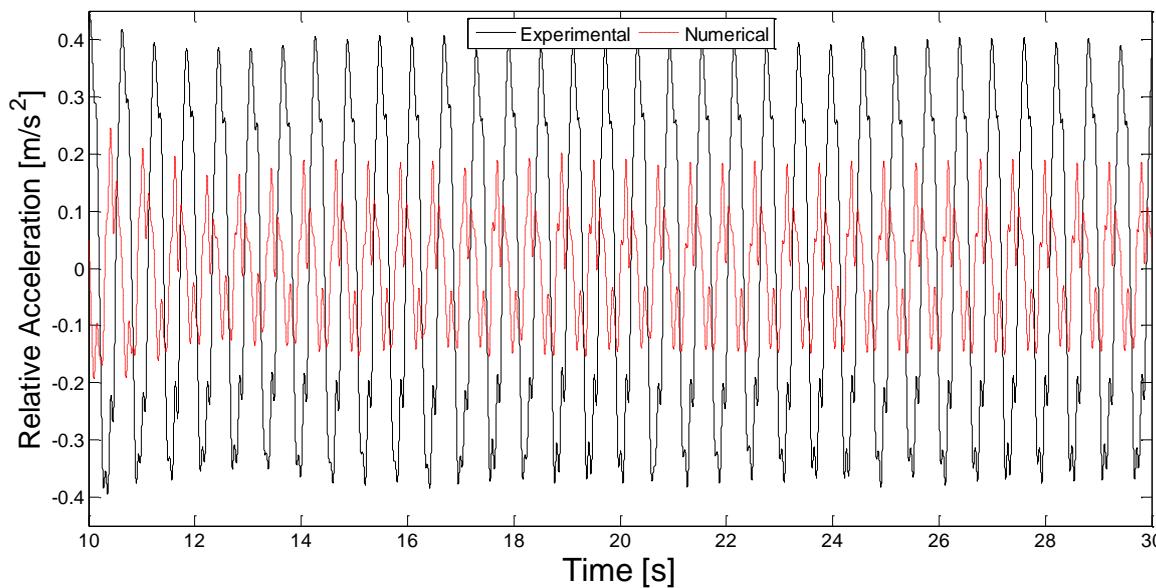
Sinusoidal test:  
 $f=1.65\text{Hz}$



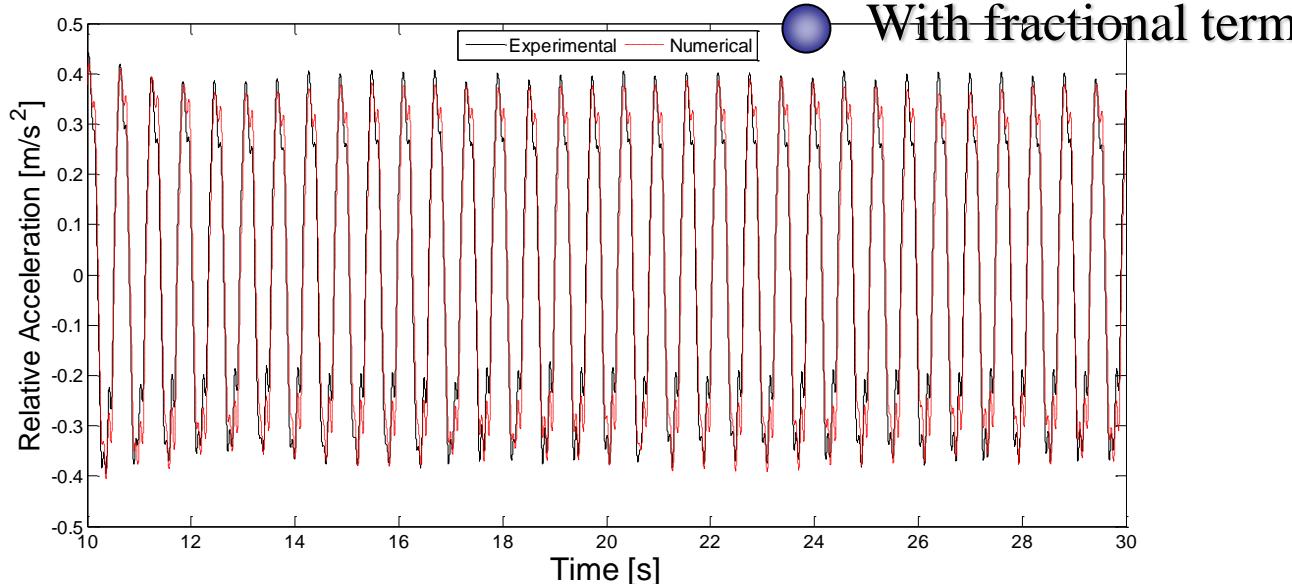
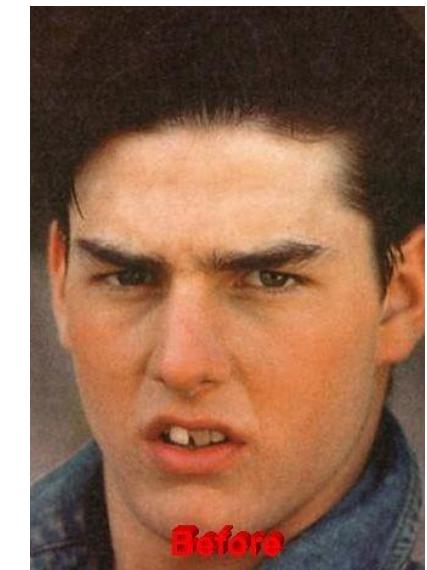
With fractional term

# Numerical Vs Experimental results

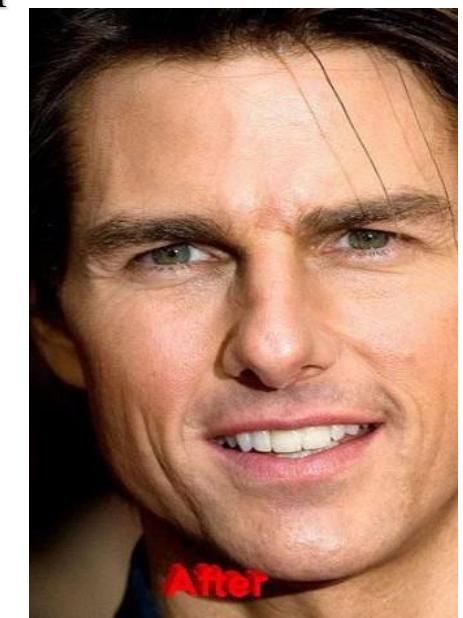
Sinusoidal test: f=1.65Hz



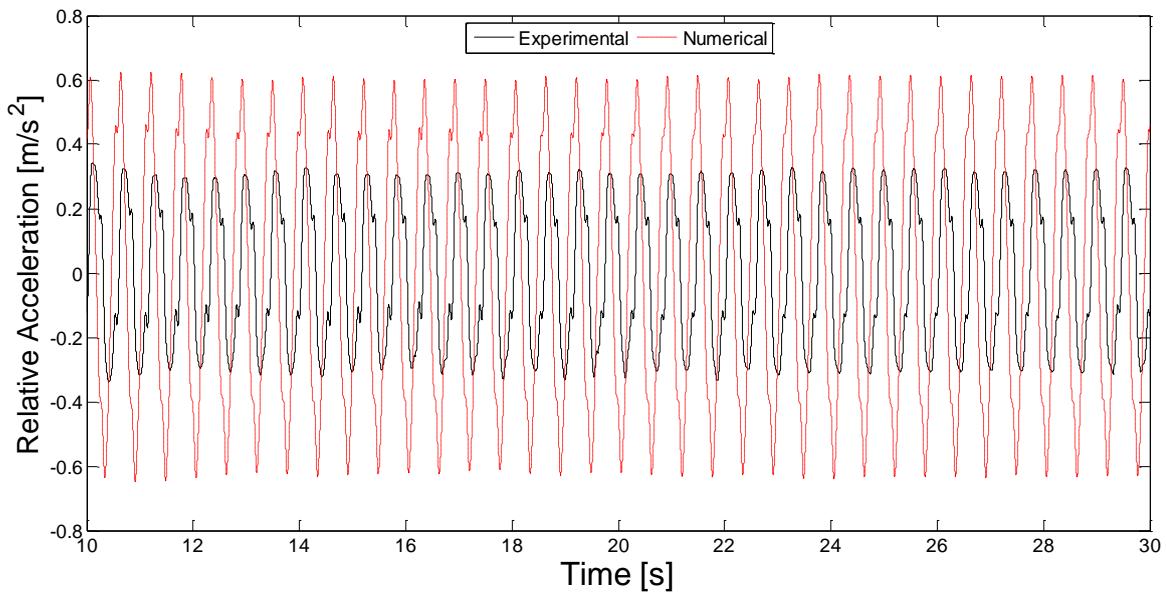
$$L_v = 5 \text{ cm}$$



With fractional term

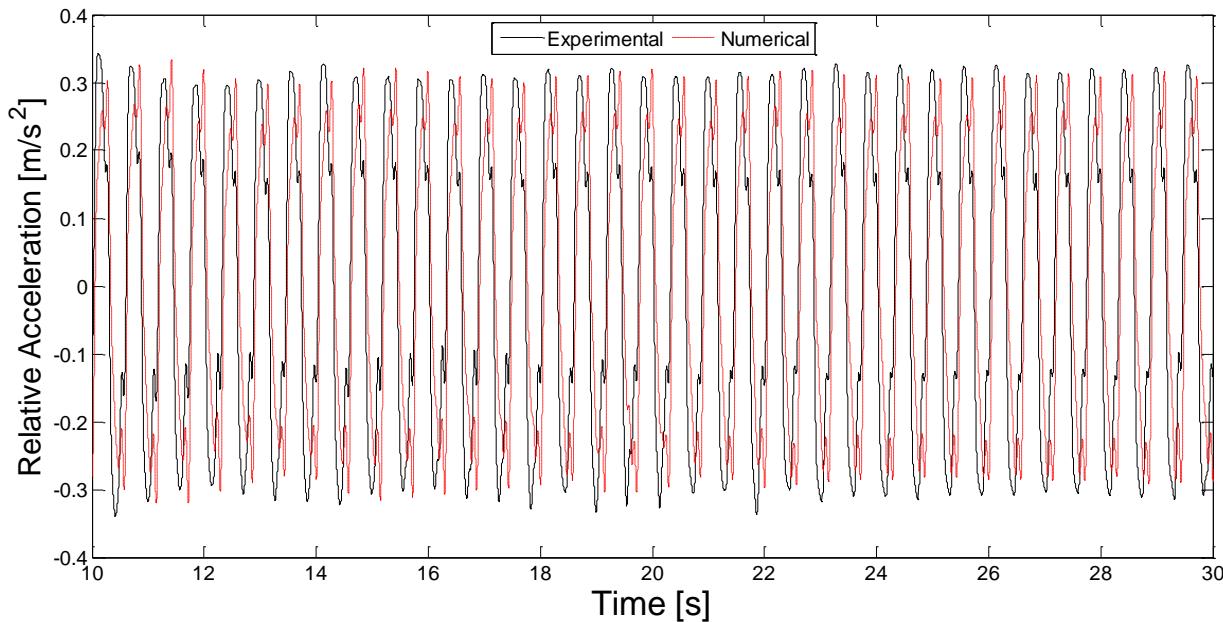


# • Numerical Vs Experimental results



Sinusoidal test:  
 $f=1.75\text{Hz}$

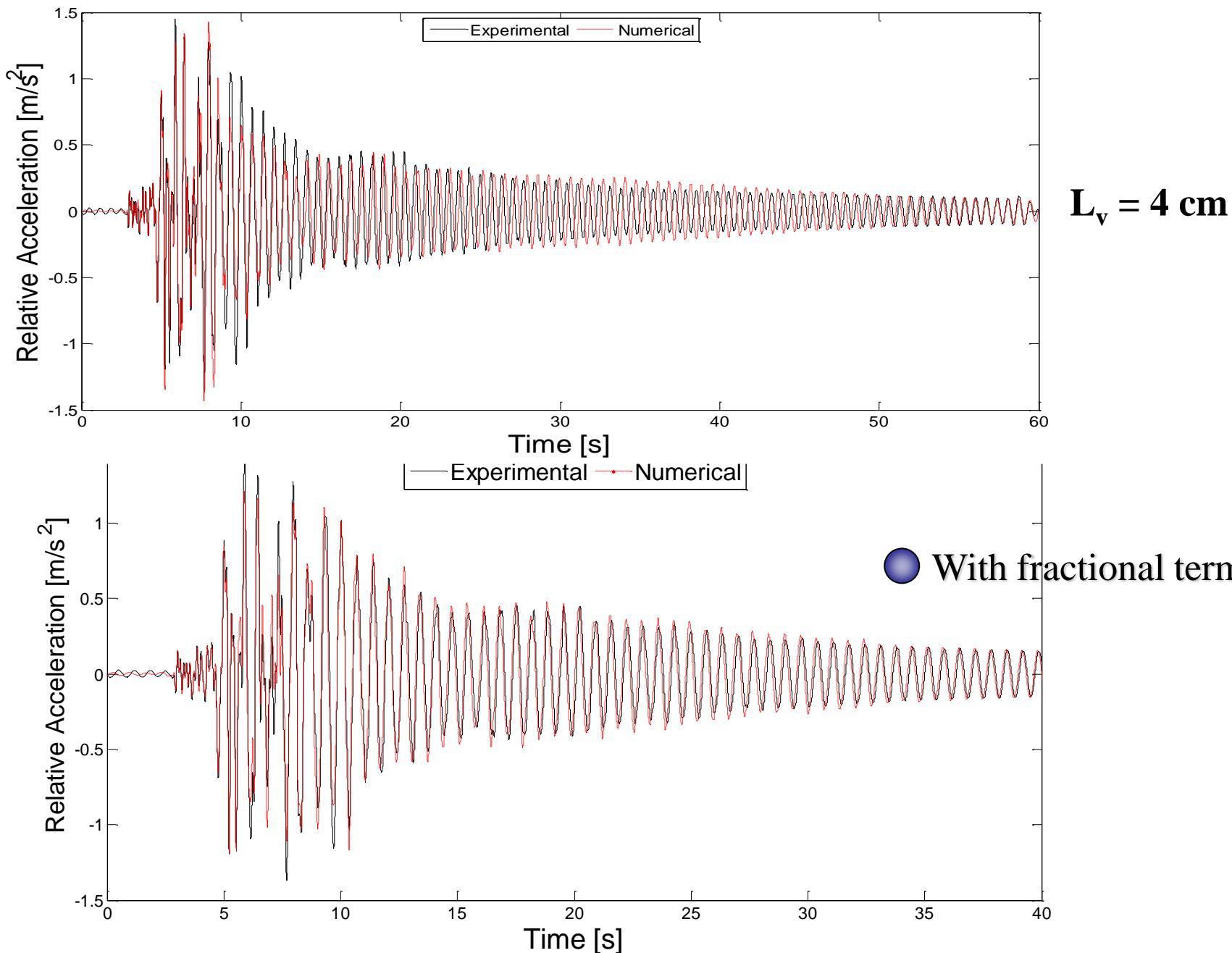
$L_v = 5 \text{ cm}$



• With fractional term

# Numerical Vs Experimental results

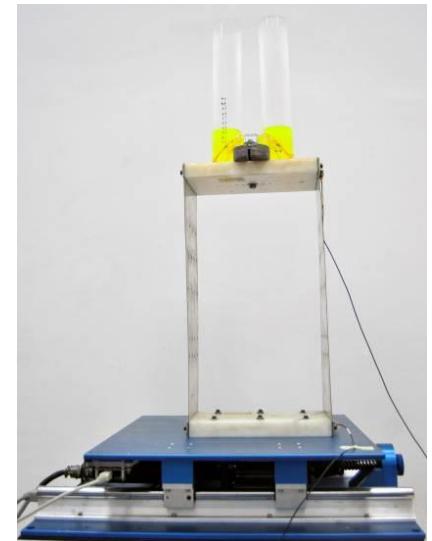
## Tolmezzo acceleration



# WE HAVE A Virtual Experiment!!!!



$$\begin{cases} (M + m_{TLC})\ddot{x} + m_h\ddot{y} + C\dot{x} + Kx = -(M + m_{TLC})\ddot{x}_g \\ m_h\ddot{x} + m_{TLC}\ddot{y} + \frac{1}{2}\rho A C_\beta \left( {}_0^C D_t^\beta y(t) \right) + 2\rho A g y = -m_h\ddot{x}_g \end{cases}$$



## Concluding Remarks

- Experimental Tests makes the difference between mathematical method and scientific method.
  - It is not easy performing tests especially for validating novel theoretical model!
- However , refining a theoretical model through tests, then allow you to have virtual experiments leading to tremendous cost savings and improved quality!
- The classical analytical formulation for TLCD allow us to predict the effectiveness of the control but, in some cases, it is not able to provide a good experimental-numerical agreement;
- An alternative formulation has been proposed to better match the experimental results then to be proposed as virtual experiment;